Time - Dependent Behaviour of SFRC Elements under Sustained and Repeated Variable Loads

Dissertation Thesis

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Summary

The research in new technologies and new materials is one of the key determinants of modern science related to civil engineering. Self-compacting concrete (SCC), high-strength concrete (HSC) etc. have opened new chapters in the use of this material in the construction industry.

One of the materials that has been used since the beginning of the last century is steel fibre reinforced concrete (SFRC). After many performed tests with straight steel fibres, it was concluded that better characteristics of composite structures were achieved with the use of different fibre geometries through an improved bond and in combination with different fibres. The main improvements achieved with the addition of fibers are the increased toughness or energy absorption capacity and the decreased crack width and deflections. With these improvements, the bearing capacity, serviceability and durability of concrete elements and structures are significantly increased.

In order to find out the influence of steel fibres and variable load on time-dependent behaviour of concrete elements, an experimental program including 24 full scale beams with cross section proportioned 15/28 cm and total length l=300 cm and 117 control specimens, was realized. The beams were manufactured from concrete class C30/37 and were divided into three series. The beams from the three series were reinforced with the same percentage of longitudinal and shear reinforcement, but with different amount of steel fibres. The first series did not contain steel fibres. In the second series, 30 kg/m³ of steel fibres (0.38% of the volume as the minimum percentage) were added, while in the third series, 60 kg/m³ of steel fibres (0.76% of the volume as the maximum dosage that did not require additional measures regarding workability) were added. The used steel fibres were hooked-end HE1/50, with an aspect ratio (length/diameter) of l/d=50, length of 50 mm, diameter of 1 mm and tensile strength of 1100 N/mm².

Regarding the loading history, the beams from each series were divided into four groups consisting of 2 beams. The beams from the first and the second group were tested under short term ultimate load at a concrete age of 40 and 400 days, accordingly. The beams from the third group were pre-cracked with service load at a concrete age of 40 days, and afterwards, a long term permanent load was applied and held up to 400 days. On the beams from the fourth group, in the considered time period of 40-400 days, a long term permanent and repeated variable load has been applied in a loading interval of 8/16 hours/day (8 hours under permanent + variable load and 16 hours under permanent load). In the case of the fourth group, an attempt has been made to simulate a realistic loading history, which is appropriate for structures such as parking garages, city bridges, warehouses, etc., where variable loads can act longer and are with a significant magnitude. At the age of 400 days, short term ultimate load testing has also been performed on the beams from the third and the fourth group.

Using the experimental results, detailed analysis of the time-dependent deformation properties of concrete and their effect on the time-dependent behaviour was carried out. For analysis of creep and shrinkage of concrete, fib Model code 2010 and B3 model (Bazant & Baweja, 2000) have been used. For the B3 model, a certain improvement and modification has been proposed.
The influence of long-term permanent and variable load on time-dependent behavior of concrete elements was obtained by the Age-Adjusted Effective Modulus Method (AAEMM) and the principle of superposition. A value for the factor of quasi-permanent value of variable action $\psi_2$ has been proposed for each type of concrete. In addition, the dependence of the factor $\psi_2$ on the variable load duration, as well as, on the ratio between the residual tensile strength and the compressive strength, is graphically presented.
Резиме

Истражувањата на нови материјали и технологии се една од главните детерминанти на денешната наука во областа на градежното инженерство. Само-вградувачкиот, високо- јакосниот, микро-армиранот и други видови бетон отвориа нови поглавија во употребата на овој градежен материјал.

Почетоците на микроармирањето со челени влакна датираат од пред еден век. По многубројните тестови и истражувања, заклучено е дека употребата на челени влакна со различни геометриски форми, како и на комбинација од нив, преку подобрување на врската со бетонот, води кон подобри карактеристики на композитната структура. Ова пред се се согледува во зголемување на жизненоста или капацитетот на абсорпција на енергија, во намалување на отворот на пукнатините и во редукција на деформациите, со што значително се подобрува носивоста, употребливоста и трајноста на бетонските елементи и конструкции.

Со цел да се утврдат влијанијата на микроармирањето, како и на променливот товар врз однесувањето на бетонските елементи во тек на време, реализирана е експериментална програма која се состоише во изработка, следење и испитување на 24 пробни елементи (греди) со напречен пресек l/d=15/28cm и должина l=300cm и 117 пробни тела за утврдување на механичко-деформациските карактерistikи на бетонот. Елементите беа изработени од класа бетон C30/37, а според начинот на армирање беа поделени во три серии. Гредите од сите серии беа армирани со иста подолна арматура и узентги, со тоа што кај оние од втората серија беа додадени 30kg/m^3 челени влакна (0.38% од волуменот, како минимален процент со кој се влијае на подобрувањето на својствата на бетонот), а кај третата серија 60kg/m^3 (0.76% од волуменот, како максимална количина која не бара дополнителни мерки за обезбедување на потребната обработливост на бетонот). Челиничните влакна се од типот HE1/50 со свитки краеви, со однос меѓу должината и дијаметарот l/d=50, (должина l=50mm и дијаметар d=1mm) и јакост на затемнување 1100 N/mm^2.

Во однос на историјата на товарење, гредите од секоја серија беа поделени на четири групи од по 2 идентични греди. Првата и втората група беа испитани до лом при старост на бетонот од 40 и 400 ден, соодветно. Каж гредите од третата група под дејство на експлоатациониот товар на старост од 40 ден беа иницирани пукнатини, по што до старост од 400 ден беа изложени само на постојан товар. Гредите од четвртата група во разгледуванит период од 40 - 400 ден беа изложени на дејство на постојан и повторуван променлив товар во циклуси од 8/16 часа / ден (8 часа дејство на постојан + променлив / 16 часа дејство на постојан). Каж оваа низа симулирана е историја на товарење која соодветствува на конструкциите каде променливите товари се задржуваат подолг временски период и се со позначителен интензитет (катни гаражи, градски мостови, магацини и сл.). На крај гредите од третата и четвртата серија беа испитани до лом.

Резиме

анализа на временски зависните карактеристики на бетонот и нивното влијание на однесувањето на елементите во тек на време. За анализа на собирањето и течењето на бетонот користени се моделот даден во fib Model code 2010 и B3 моделот (Bazant & Baweja, 2000) за кој, врз основа на експериментално добиените резултати, е предложено одредено подобрување.

Определувањето на влијанието на долготрајните дејства, односно на постојаниот и променливиот товар, на однесувањето на бетонските елементи е спроведено со помош на Методот на корегиран ефективен модул на еластичност (AAEMM) и принципот на суперпозиција. За секој од трите разгледувани случаи предложена е пооделна вредност за коефициентот за дефинирање на квази – перманентната вредност на променливите товари $\psi_2$ и во графички облик е дефинирана неговата зависност од временетраењето на променливиот товар и зависноста од односот помеѓу остаточната јакост на затепнување и јакоста на притисок на бетонот.
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CHAPTER 1: INTRODUCTION

1.1 Scope and Objective of the Research

The increased complexity of modern structures with unique geometries, high-rise and supertall buildings, large-span cable-stayed or prestressed bridges, extremely thick or thin structures etc., needs special attention regarding their time-dependent behaviour.

Having in mind the positive influence of steel fibers on the general behavior of structures, they have very often been added to the standard or special concrete mixtures in some most recent applications (Figure 1.1). They increase the toughness and ductility of structures and are known to contribute to deflection and crack width control.

In the CCTV (China Central Television) tower in Beijing, steel fibres were used together with ordinary reinforcement and were incorporated in self-compacting concrete. The open roof building “Maison de l’écriture” in Switzerland, which was designed as an inspiring place for writers, also consists of steel fibre reinforced concrete. To guarantee the durability of the floors of the “Gardens by the Bay” structure in Singapore, which were around 1 million m², SFRC was used. In the Oceanographic Park in Valencia, the unique thin shell structure was designed by a combination of mesh reinforcement and steel fibres.

Figure 1.1: Structures made of SFRC by Bekaert: CCTV tower (up left), “Maison de l’écriture” (up right), Oceanographic Park (down left) and “Gardens by the Bay” (down right)
An appropriate evaluation of creep and shrinkage effects and their influence on structural reliability requires a rational approach with respect to two problems that are interrelated, but are frequently considered in the design practice independent and dealt with separately:

- Prediction of creep and shrinkage strains (a material properties problem),
- Determination of the related time-dependent structural response (a structural analysis problem) [7].

Knowing the time-dependent properties of any type of concrete is very important if we want to use it as a structural material. This type of research needs serious financial resources, long-term occupation of a laboratory and a lot of time for testing and measurement. That is why there are not many research studies dealing with this subject, namely steel fibre reinforced concrete.

Deflections and crack widths predicted on the basis of short-term tests do not provide satisfactory results for verification in the serviceability limit states. That is why long-term experiments of reinforced concrete elements under sustained loads are very important. Long term sustained load causes a significant increase of deflections and crack widths. In addition, long-term variable repeated load causes additional increase. This is proven by the experimental and theoretical research that has been carried out at the Faculty of Civil Engineering – Skopje for the last 12 years. Following the original idea of Prof. Atanasovski [2], the effect of realistic load histories on the creep and structural behavior was studied in several research projects. Prof. Markovski studied the influence of variable loads on time-dependent behavior of prestressed concrete elements [21], while Doc. Arangjelovski studied the time-dependent behavior of reinforced high-strength concrete elements under the action of variable loads [1].

Verification of the serviceability limit states in Eurocodes should be done according to three combinations of actions, taken into account in relevant design situations: characteristic, \( G_{k,j} + P + Q_{k,1} + \psi_{0,i}Q_{k,i} \); frequent \( G_{k,j} + P + \psi_{1,1}Q_{k,1} + \psi_{2,i}Q_{k,i} \) and quasi – permanent combination \( G_{k,j} + P + \psi_{2,i}Q_{k,i} \) [12]. The quasi – permanent combination is the one that is used for long – term effects. The recommended values for the factor of quasi-permanent value of variable action \( \psi_{2} \) for reinforced concrete structures are presented in Table 1.1. These values can also be set in the National Annex [12].

To define the influence of variable repeated load on long-term behavior of SFRC elements, a method of replacement of the variable load with quasi-permanent load determined by the factor \( \psi_{2} \) will be used, i.e. part of the variable load will act as a permanent load [12].

Several studies have been conducted on long-term behavior of SFRC beams under sustained loads, while studies which include the effect of variable repeated load are uncommon.

The aim of this research has been to contribute to the increase of the data base on SFRC regarding the effects of creep and shrinkage, evaluate the obtained results with the most advanced and latest codes in this area and propose a value for the factor \( \psi_{2} \) for the studied types of concrete and variable load duration of 8 hours.
Table 1.1: Recommended values for the factor $\psi_2$ [12]

<table>
<thead>
<tr>
<th>Action</th>
<th>Factor $\psi_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imposed loads on buildings, category</td>
<td></td>
</tr>
<tr>
<td>Category A: domestic, residential areas</td>
<td>0.3</td>
</tr>
<tr>
<td>Category B: office areas</td>
<td>0.3</td>
</tr>
<tr>
<td>Category C: congregation areas</td>
<td>0.6</td>
</tr>
<tr>
<td>Category D: shopping areas</td>
<td>0.6</td>
</tr>
<tr>
<td>Category E: storage areas</td>
<td>0.8</td>
</tr>
<tr>
<td>Category F: traffic area, vehicle weight $\leq$30kN</td>
<td>0.6</td>
</tr>
<tr>
<td>Category G: traffic area, 30kN$&lt;$ vehicle weight $\leq$160kN</td>
<td>0.3</td>
</tr>
<tr>
<td>Category H: roofs</td>
<td>0.0</td>
</tr>
<tr>
<td>Snow loads on buildings, countries</td>
<td></td>
</tr>
<tr>
<td>Finland, Iceland, Norway and Sweden</td>
<td>0.2</td>
</tr>
<tr>
<td>Remainder of CEN Members, for altitude $H&gt;1000$ m a.s.l.</td>
<td>0.2</td>
</tr>
<tr>
<td>Remainder of CEN Members, for altitude $H \leq$1000 m a.s.l.</td>
<td>0.0</td>
</tr>
<tr>
<td>Wind loads on buildings</td>
<td>0.0</td>
</tr>
<tr>
<td>Temperature (non-fire) in buildings</td>
<td>0.0</td>
</tr>
<tr>
<td>Vertical traffic actions in road bridges, footbridges and railway bridges</td>
<td>0.0</td>
</tr>
</tbody>
</table>

1.2 Outline of the Thesis

The thesis is organized in 9 Chapters. It contains 98 equations, 34 tables and 136 Figures. It starts with Chapter 1, where the scope and the objective of the research are underlined.

Chapter 2 is intended to provide general knowledge about the idea of using fibers in concrete, or more specifically steel fibers. It shows the state of the art regarding fiber and steel fiber reinforced concrete, emphasizing the specifics of the mix design, as well as the main characteristics of the materials and the influence of fibres on the physical and mechanical properties of concrete. At the end of this chapter, current developing technologies in this area are presented.

Chapter 3 deals with concrete strain components, when concrete is subjected to sustained loads. It contains a detailed explanation of the time-dependent deformation properties, such as creep and shrinkage.

Chapter 4 contains a literature review of some more important investigations dealing with creep and shrinkage of steel fiber reinforced concrete, as well as time-dependent structural response of SFRC beams. Results from certain long-term investigations of SFRC in terms of long-term deflections and long-term crack widths are also presented.

The next chapter deals with the constitutive law or stress-strain relations for SFRC and reinforcement. It presents the stress-strain relation under compression and tension, pointing out the differences between ordinary and steel fiber reinforced concrete. It continues with the stress-strain relation under long-term load and provides an overview of two models which can be
considered as the most advanced and latest actual models for design of creep and shrinkage: fib Model code 2010 and B3 model. These codes were later used in the analytical analysis presented in chapter 8.

Chapter 6 points out the available methods for structural analysis of creep and provides an overview only of the method that has been used in the thesis, namely the Age-Adjusted Effective Modulus Method (AAEMM). The application of AAEMM for steel fibre reinforced concrete that contains plain reinforcement is also discussed in this chapter.

In Chapter 7, the experimental program is explained in detail. It contains description of the experimental program, information about the application of the load in the specific loading histories, description of the used measurement technique and results from the testing of mechanical and time-dependent deformation properties of each type of concrete. The results referring to the effects of long-term loading on deflections, crack widths and strains of the beams and comparison between the different types of concrete are presented at the end of this chapter.

The analytical analyses of the results are presented in Chapter 8. The experimental results are compared with the analytical results. The analytical analysis of creep and drying shrinkage was performed by use of the B3 model and fib Model code 2010. In the beginning, the analyses were done only for the time period considered in this research, namely, 400 days. Based on these results, the analyses according to the previously mentioned models were prolonged up to age of service life of structures of 100 years. Data from these analyses were later used to calculate the time-dependent deflections using the Age-Adjusted Effective Modulus Method. Quasi-permanent load procedure and the principle of superposition were used to obtain the value of the factor $\psi_2$.

The main conclusions are presented in Chapter 9.

At the end of the thesis, the references are stated.
CHAPTER 2: STATE OF THE ART - FRC and SFRC

2.1 Fibre Reinforced Concrete

Fibre reinforced concrete (FRC) is concrete made primarily of hydraulic cements, aggregates, and discrete reinforcing fibers. The concrete matrices may be mortars, normally proportioned mixes, or mixes specifically formulated for a particular application [24].

2.1.1 Historical Aspects

Composite materials, reinforced with fibrous materials have been used since ancient times. Approximately 3500 years ago, straw was used to reinforce sun-baked bricks [5], and horsehair was used to reinforce masonry mortar. The first widely commercial use of fibres was the use of asbestos fibres in a cement paste matrix in the year of 1898, which began with the invention of the Hatschek process. Since the beginning of the last century, at various intervals, short pieces of steel have been included within concrete in an attempt to improve its strength, ductility, durability and to overcome the typical characterization of brittleness of the cementitious materials. However, there was not much interest by research organizations or by the construction industry until the year of 1963, when Romualdi and Batson published the results of an investigation carried out on steel fibre reinforced concretes [11]. In the beginning, it was assumed that short pieces of steel or steel fibres enhance mostly the tensile strength of composites. In the year of 1964, Broms and Shah systematically studied the mechanical properties of the new material. Since then, the interest for this material has been growing very quickly. Intensive investigations began not only for the mechanical characteristics of the steel fibre reinforced concrete, but also for the determination of the influence of the steel fibres on the behaviour of different concrete and reinforced concrete elements. It comes out that the major contribution of the steel fibres is in enhancing the toughness and the durability of the elements.

2.1.2 Type of Fibers

Through the developing years of this material, a wide variety of fibres have been used with hydraulic cements: steel, glass, asbestos, carbon, aramid, synthetic and natural fibres [5].

Generally, there are two groups of fibres.

The first group is consisted from high modulus and high strength fibres like steel and glass fibres which are, the so called, conventional fibres, and relatively new fibres such as carbon (high strength), aramid (kevlar) and polyvinyl alcohol fibres. These fibres produce strong composites. Besides the energy absorption capacity, they have an influence on the strength and stiffness of the composites. Steel fibres are used in road and airport pavements, in industrial floors, in thin wall precast elements, in shotcrete applications for ground support, rock slope stabilization and tunneling, in dams, bridge deck slabs, water pipes, railway sleepers, as well as in many structural and repair applications to replace secondary reinforcement and stirrups or to act together with the primary conventional reinforcement. On the other hand, glass fibres are mainly used for architectural exterior facade panels due to their light weight and low-cost [5].
The second group is consisted from low modulus and high elongation fibres like synthetic or organic polymers, man made fibres (polypropylene, polyethylene, polyester, nylon, acrylic) and natural (cellulose, sisal, jute). These fibres do not lead to strength improvement but produce composites with big energy absorption capacity, increased toughness and resistance to impact and explosive loading. Synthetic fibres are also used for the control of shrinkage cracks [5].

Asbestos fibres were widely used in the past, but due to health hazards associated with all asbestos products, new types of fibres were developed [5].

All these different types of fibres vary in respect to their mechanical characteristics, their efficiency and cost. Consequently, the properties of fiber reinforced concrete depend not only on the matrix properties, but also on the fibre type, fibre geometry and fibre dosage.

Hence, the choice of the right fibre reinforced concrete for a specific purpose depends on which characteristics we want to improve. Some of the main properties of different fibres are summarized in Table 2.1.

<table>
<thead>
<tr>
<th>Fibre</th>
<th>Diameter (μm)</th>
<th>Specific gravity</th>
<th>Modulus of Elasticity (GPa)</th>
<th>Tensile Strength (GPa)</th>
<th>Elongation at break (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>5-500</td>
<td>7.84</td>
<td>200</td>
<td>0.5-2.0</td>
<td>0.5-3.5</td>
</tr>
<tr>
<td>Glass</td>
<td>9-15</td>
<td>2.6</td>
<td>70-80</td>
<td>2.0-4.0</td>
<td>2.0-3.5</td>
</tr>
<tr>
<td>Asbestos</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crocidolite</td>
<td>0.02-0.4</td>
<td>3.4</td>
<td>196</td>
<td>3.5</td>
<td>2.0-3.0</td>
</tr>
<tr>
<td>Chrysolite</td>
<td>0.02-0.4</td>
<td>2.6</td>
<td>164</td>
<td>3.1</td>
<td>2.0-3.0</td>
</tr>
<tr>
<td>Polypropylene</td>
<td>20-400</td>
<td>0.9-0.95</td>
<td>3.5-10</td>
<td>0.45-0.76</td>
<td>15-25</td>
</tr>
<tr>
<td>Aramid (kevlar)</td>
<td>10-12</td>
<td>1.44</td>
<td>63-120</td>
<td>2.3-3.5</td>
<td>2.0-4.5</td>
</tr>
<tr>
<td>Carbon</td>
<td>8-9</td>
<td>1.6-1.7</td>
<td>230-380</td>
<td>2.5-4.0</td>
<td>0.5-1.5</td>
</tr>
<tr>
<td>Nylon</td>
<td>23-400</td>
<td>1.14</td>
<td>4.1-5.2</td>
<td>0.75-1.0</td>
<td>16.0-20.0</td>
</tr>
<tr>
<td>Cellulose</td>
<td>-</td>
<td>1.2</td>
<td>10</td>
<td>0.3-0.5</td>
<td>-</td>
</tr>
<tr>
<td>Acrylic</td>
<td>18</td>
<td>1.18</td>
<td>14-19.5</td>
<td>0.4-1.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Polyethylene</td>
<td>25-1000</td>
<td>0.92-0.96</td>
<td>5</td>
<td>0.08-0.60</td>
<td>3-100</td>
</tr>
<tr>
<td>Wood fibre</td>
<td>-</td>
<td>1.5</td>
<td>71</td>
<td>0.9</td>
<td>-</td>
</tr>
<tr>
<td>Sisal</td>
<td>10-50</td>
<td>1.5</td>
<td>-</td>
<td>0.8</td>
<td>3.0</td>
</tr>
<tr>
<td>Cement matrix</td>
<td>-</td>
<td>1.5-2.5</td>
<td>10-45</td>
<td>0.003-0.007</td>
<td>0.02</td>
</tr>
</tbody>
</table>

In the draft European Standard prEN 14889-1:2004 [23], fibres are divided into classes in accordance with the intended use:

- Class I; intended primarily to improve the short term plastic properties of mortar and/or concrete by controlling plastic shrinkage, settlement cracks and reducing bleeding, but not adversely affecting the long term properties.
2.2 Steel Fibre Reinforced Concrete

Steel fibre reinforced concrete (SFRC) is concrete made of hydraulic cements containing fine or fine and course aggregates and discontinuous discrete steel fibers [24].

2.2.1 Steel Fibres

In the draft European Standard prEN 14889-1:2004 [23], the following definition is provided for steel fibres: “Steel fibres are straight or deformed pieces of cold-drawn steel wire, straight or deformed cut sheet fibres, melt extracted fibres, shaved cold drawn wire fibres and fibres milled from steel blocks which are suitable to be homogeneously mixed into concrete or mortar”. According to this standard, steel fibres are divided into five general groups:

- Group I, cold-drawn wire;
- Group II, cut sheet;
- Group III, melt extracted;
- Group IV, shaved cold drawn wire and
- Group V, milled from blocks [23].

Steel fibres can also have coatings, like zinc coating for improved corrosion resistance or brass coating for improved bond characteristics.

The steels used for making fibres are generally carbon steels or stainless steels, which are primarily used for corrosion-resistant fibres, in refractory applications and in marine structures.

It is necessary that the fibres have high tensile strength, so that they are pulled out, but not broken. If they are broken, brittle failure can occur as in plain concrete. Therefore ASTM A 820 established the minimum tensile yield strength of 345 MPa, while JSCE Specification requirement is 552 MPa [24].

The Japanese Society of Civil Engineers (JSCE) has classified steel fibres based on the shape of their cross section, which depends on the manufacturing method [24]:

- Type 1, Square section;
- Type 2, Circular section, and
- Type 3, Crescent section.

The fibres differ also regarding their shape along length. In the first performed experiments, only straight steel fibres were used. They did not develop a sufficient bond with the concrete matrix and high dosages were needed to increase toughness. These high dosages resulted in...
problems with workability and fibre balling due to the high aspect ratio (length to diameter ratio). That is why the fibres that are used today are with different deformed shape and rough surfaces, with hooked or enlarged ends, crimped or with irregular shape. The problem with fibre balling remained and to avoid this problem, the fibres can be collated into bundles of 10-30 fibres using water-soluble glue. In this way, they are dispersed in the concrete, the glue is dissolved and they can be randomly distributed.

Also, it is very important that fibres must have length that is at least 2-3 times the maximum aggregate size. If this is not the case, there is no good crack bridging effect and the fibres act as aggregates [10].

Some typical fibre geometries as well as fibre cross section shapes are presented in the next figure.

![Fibre cross section shapes and some typical fibre geometries](image)

Figure 2.1: Fibre cross section shapes and some typical fibre geometries

### 2.2.2 Mix Design of SFRC

For relatively small fibre volumes of up to 0.5%, the conventional mix designs for plain concrete may be used without any modification. For larger fibre volumes, due to the decreased workability, mix design procedures that emphasize workability should be used [5].

One of the main factors that mostly affects workability is the aspect ratio (l/d) of the fibres. The recommended values of the aspect ratio are in the range of 20-100 because workability decreases with increasing of the aspect ratio [11]. Edgington et. al. also reported that the workability decreased as the size and quantity of the aggregate particles larger than 5mm was increased [11]. They proposed an equation by which they estimated the critical percentage of
fibres which would make the SFRC unworkable. To ensure proper compaction, the fibre content should not exceed 0.75 \( PWC_{\text{crit}} \).

\[
PWC_{\text{crit}} = 75 \cdot \frac{\pi \cdot SG_f \cdot d}{SG_c} \cdot \frac{d}{l} \cdot K \]  

(2.1)

Where:

- \( PWC_{\text{crit}} \) is the critical percentage of fibres, by weight of the concrete matrix;
- \( SG_f \) is specific gravity of fibres;
- \( SG_c \) is specific gravity of concrete matrix;
- \( d/l \) is inverse of the fibre aspect ratio and
- \( K = W_m/(W_m+W_a) \), where \( W_m \) is weight of the mortar fraction (particle size smaller than 5mm) and \( W_a \) is the weight of the aggregate fraction (particle size larger than 5mm).

When compared to plain concrete, SFRC mixes contain higher cement contents and higher fine to coarse aggregate ratio.

Some typical range of proportions for normal weight SFRC, depending on the maximum aggregate size, are given in Table 2.2.

Table 2.2: Range of proportions for normal weight SFRC [24]

<table>
<thead>
<tr>
<th>Maximum aggregate size</th>
<th>9.5 mm</th>
<th>19 mm</th>
<th>38 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement (kg/m³)</td>
<td>355–600</td>
<td>300–535</td>
<td>280–415</td>
</tr>
<tr>
<td>W/c</td>
<td>0.35–0.45</td>
<td>0.35–0.50</td>
<td>0.35–0.55</td>
</tr>
<tr>
<td>Fine/coarse agg. (%)</td>
<td>45–60</td>
<td>45–55</td>
<td>40–55</td>
</tr>
<tr>
<td>Entrained air (%)</td>
<td>4–8</td>
<td>4–6</td>
<td>4–5</td>
</tr>
<tr>
<td>Fibre content (%) by volume</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Smooth fibres</td>
<td>0.8–2.0</td>
<td>0.6–1.6</td>
<td>0.4–1.4</td>
</tr>
<tr>
<td>- Deformed fibres</td>
<td>0.4–1.0</td>
<td>0.3–0.8</td>
<td>0.2–0.7</td>
</tr>
</tbody>
</table>

As it can be noticed from the previous table, there are also minimum amounts of fiber content that should be added to the concrete mixture in order to have some significant increase in toughness, as crucial and most important property of SFRC. As proven with some tests, if very small amount of fibres is used, there is a high probability that some regions in the concrete mixture will remain unreinforced and brittle failure can occur. Therefore, the usual minimum amount of fibres is 20-25-30 kg/m³, while the maximum amount that will not require special workability demands is 70-80 kg/m³.

Based on their long experience in producing and mixing SFRC, the biggest world manufacturer of steel fibres, Bekaert – Belgium, gives some important recommendations that each engineer dealing with SFRC should know:

- The optimum slump before adding the fibres should be greater than 12cm;
- Fibres should be added in the plant mixer together with sand and aggregate or should
be added to the fresh mixed concrete, but never as the first component;
• Fibres should be added continuously, at a maximum speed of 40kg/min;
• After adding all fibres to the mixer, the mixing time should be 1 min/m³ concrete and not less than 5 min.

2.2.3 Main Characteristics of SFRC
When we speak about SFRC, we assume that the fibres are uniformly randomly oriented in the concrete matrix. However, after placing and vibrating, no one can be sure that this is true. This often leads to a large scatter in the test data and high variability in measured values due to the direction of loading in relation to the direction of casting.

When table vibration or excessive internal vibration is used, fibres tend to become horizontally aligned. They also show preferential parallel alignment close to the bottom and sides of the molds. With electromagnetic measurements and X-ray photographs of the fibre distribution, it was proven that fibres showed not only preferential alignment, but also non-uniform distribution along the length of the SFRC beam. Therefore, small amounts of fibre reinforcement, less than 30 kg/m³, should not be used because they lead to more non-uniform distributions. On the other hand, the preferential alignment can be good, if the fibres can be oriented in the direction of the acting stress. However, if all recommendations for mix design, mixing, handling, placing and finishing are followed, it is possible to produce SFRC with acceptably low variability in the fibre distribution and orientation [5].

The effectiveness of the fibres in improving the characteristics of concrete is dependent on the fibre matrix interactions, which are governed by the closer zone around the fibres, called interfacial transition zone (ITZ). This zone, just around the fibre, is significantly different from the other zones of the matrix. When fibres are added to the concrete, they act like special long aggregates that are preventing the aggregate to fulfill the spaces between other aggregates. In this way, there is more cement paste around the fibres to fill the empty space (wall effect) [10]. The three main fibre matrix interactions are [5]:
• Physical and chemical adhesion;
• Friction, and
• Mechanical anchorage induced by deformations on the fibre surface or by overall complex geometry.

Physical and frictional bondings between a steel fibre and a cementitious matrix are very weak and therefore mechanical anchoring is required.

Fibres actually act through stress transfer from the matrix to the fibre by some combination of interfacial shear and mechanical interlock between the deformed fibre and the matrix. Up to the point of matrix cracking, the load is carried by both the matrix and the fibres. When cracking occurs, the fibres carry the entire stress by bridging across the cracked concrete until they pull out completely. Actually, the energy is dissipated as the fibre undergoes plastic deformation while being pulled out. Many models were developed to describe the pull out behavior of hooked-end fibers. In the model of Alwan et. al. (Figure 2.2) the contribution of deformation is simulated by the
formation of a plastic hinge, which is generated as the fibre slips during the pull-out process [5].

- at the onset of complete debonding

- mechanical interlock with two plastic hinges

- mechanical interlock with one plastic hinge

- at onset of frictional bond.

Figure 2.2: Pull-out behavior of hooked-end steel fibre when being extracted from the matrix [5]

The resistance of the fibres to pull-out increases with the increasing of the aspect ratio and the bond strength increases with the increasing of the matrix and fibre strength. However, the post-cracking behavior mainly depends on the fibre geometry and therefore, there are new products on the market (as the one from the Bekaert company) with increased number of hooks and improved material, as presented in Figure 2.3.
2.2.4 Physical and Mechanical Properties of SFRC

2.2.4.1 Modulus of Elasticity and Poisson’s Ratio

When the volume percentage of fibres is less than 2%, the Modulus of Elasticity and the Poisson’s ratio of SFRC can be taken as equal to those of plain concrete [24].

2.2.4.2 Compressive Strength

The compressive strength is only slightly affected by the presence of fibers. The observed increases range from 0-15% for up to 1.5% fibers by volume, as reported in [24], or from 0-25% for up to 2% fibers by volume [5]. Although strength is not significantly increased, the energy absorption (post-cracking ductility) of the material under compression is improved [5].

2.2.4.3 Tensile Strength

The direct tensile strength is significantly increased for 30-40% with the addition of 1.5% fibres by volume [24], or 0-60% [5]. This is valid for more or less randomly distributed fibres, while for fibres aligned in the direction of the tensile stress, the increase can be 133% for 5% smooth straight steel fibres by volume [5].

2.2.4.4 Flexural Strength

The flexural strength is much more improved by the addition of steel fibres. 50-70% increase has been reported using the usual fibre volume and standard third-point bending test [24]. The increase can be 100% or even 150% if bigger fibre volumes are used, if center-point bending test is performed or if smaller specimens are used. Except the fibre volume, the shape of the fibers and the aspect ratio also plays a crucial role, with deformed fibres and bigger aspect ratio being more effective [5].

2.2.4.4 Shear and Torsion

The improvement in the residual strength of the concrete elements with the addition of steel fibres leads to an increase of shear capacity. For 1% steel fibres by volume, the shear strength was increased for 0-30% [24]. There is not much data about torsional strength, but different studies also show an increase ranging from 0-100% [5].
2.2.4.5 Toughness

The biggest improvement that is achieved by addition of steel fibres to plain concrete is in the energy absorption capacity or in the toughness. The flexural toughness can be defined as the area under the complete load-deflection curve or it is the total energy needed for a fracture. It increases with bigger fibre dosage, bigger aspect ratio and with the use of deformed fibers.

2.2.4.6 Behavior under Impact Loading

For normal strength concrete under flexural impact load, the peak loads for SFRC were about 40% higher than those for plain concrete. The fracture energy, which is the second most important parameter when observing impact loading, was increased for 2.5 times [24].

2.2.4.7 Fatigue

As fibres do not increase significantly the static compressive strength, there is also no significant improvement in the fatigue strength under compressive loading. On the other hand, as it is the case with the static tensile strength, fibres increase the fatigue strength under tensile loading. Steel fibres enable higher endurance limits, finer cracks and more energy absorption to failure [5].

2.3 Developing Technologies

The most commonly used fibre concrete mixes can be considered as tension softening materials. This means that after cracking of the matrix, the fibres still carry tensile stress across the crack, but this post-cracking tensile stress is smaller than the tensile strength of the matrix. When using steel fibre reinforced concrete, tension softening material can be obtained by adding up to 1% steel fibres from the volume or 78.5 kg/m$^3$. In this case, fibres are usually added to the concrete during mixing.

A developing technology is a material called SIFCON or Slurry Infiltrated Fibre Concrete. It can contain up to 10% steel fibres from the volume or 785 kg/m$^3$ (even 25% have been reported) and it is a strain hardening material, which means that the post-cracking tensile stress is higher than the cracking tensile strength. The resulting composite material has high strength and ductility, but needs special casting techniques to be produced. The fibres in this case are pre-placed into an empty mould and high strength cement-based slurry consisting of very fine particles is infiltrated subsequently. Compressive strengths from 21 – 140 MPa have been achieved with the use of additives like fly ash, micro silica and admixtures. Tensile strengths of up to 41MPa with tensile strains close to 2% and shear strengths of up to 28MPa have been reported. The compressive toughness was increased 50 times with strain capacity of more than 10%, while the tensile toughness was increased 1000 times when compared to unreinforced concrete. SIFCON is being developed for military applications, such as hardened missile silos, but can also be used for impact and blast resistant structures, as refractories or heat resistant materials, revetments, pavement repairs and for public sector applications, such as energy absorbing tanker docks [24].

One of the recent developments is the so called ECC or Engineered Cementitious Composites. It is also called bendable concrete. It is a composite reinforced with specially
selected short random, usually polymer fibres. It is a strain hardening material obtained with only about 2% steel fibres from the volume and it does not contain coarse aggregate. It is characterized by very fine distributed cracking pattern due to the fact that the matrix and the fibre dosages are determined so that the load needed to pull out the fibres is bigger than the cracking load [10].

One of the most promising materials is HFRC or Hybrid Fiber Reinforced Concrete. It contains two or more different fibre types (hybrid reinforcement) that are mixed so that the overall material is optimized to achieve synergy. The overall performance of the composite exceeds the performance induced by each of the fibres alone [5]. The synergies were classified by Banthia and Gupta into three groups, depending on the mechanisms involved [5]:

- Hybrids based on fibre constitutive response, where one fibre is stronger and stiffer and provides strength, while the other is more ductile.
- Hybrids based on fibre dimensions where one fibre is small (micro or mesofibre) and provides microcrack control at earlier stages of loading to arrest microcracks and enhance the first crack and strength, while the other fibre, which is bigger (macrofiber), provides the bridging mechanisms across macrocracks and induces toughness at high strains and crack openings.
- Hybrids based on fibre function where one type of fibre induces strength or toughness in the hardened composite, while the second type of fibre provides fresh mix properties suitable for processing.

The fibres used in HFRC can be made either from one material, but with different geometries, or can be composed of different materials, such as polyethylene microfibers for microcrack control and deformed steel fibres for macrocrack bridging [5].

Another developing material, which incorporates steel fibres, as it is the case with the High Strength Concrete (HSC), is the Reactive Powder Concrete (RPC). Flexural strengths can reach 60 MPa with 2.4% steel fibres by volume or even 102 MPa with 8% steel fibres by volume. The used fibres are with approximate length of 15mm and diameter of 0.2mm [5].

All these developing new materials can be classified as High Performance Fibre Reinforced Cement Composites (HPFRCC) or Ductile Fibre Reinforced Cement Composites (DFRCC) and it seems that the future of concrete as a material is in the use of these high performance systems. The only negative aspect is that the use of high fibre volume is expensive, but the ECC proves that high performance systems can be obtained also with smaller fibre volumes.
CHAPTER 3: CONCRETE STRAIN COMPONENTS

3.1 General
In general, total concrete strain $\varepsilon_c(t)$ at any time $t$, in an uncracked, uniaxially-loaded concrete element, at time $t_0$ ($t>t_0$), under constant stress $\sigma_c(t_0)$, can be presented as a sum of the separate strains:

$$\varepsilon_c(t) = \varepsilon_{ci}(t_0) + \varepsilon_{cc}(t) + \varepsilon_{cs}(t) + \varepsilon_{cT}(t) \ [14]$$

\[3.1\]

If temperature is kept constant:

$$\varepsilon_c(t) = \varepsilon_{ci}(t_0) + \varepsilon_{cc}(t) + \varepsilon_{cs}(t) \ [14]$$

\[3.2\]

Where:
- $\varepsilon_c(t)$ – is the total strain in concrete at time (t);
- $\varepsilon_{ci}(t_0)$ – is instantaneous strain in concrete at loading at time (t_0);
- $\varepsilon_{cc}(t)$ – is strain from concrete creep at time (t);
- $\varepsilon_{cs}(t)$ – is strain from shrinkage at time (t);
- $\varepsilon_{cT}(t)$ – is strain from temperature or thermal strain at time (t).

The equation 3.1 can also be written in the following form:

$$\varepsilon_c(t) = \varepsilon_{c\sigma}(t) + \varepsilon_{cn}(t) \ [14]$$

\[3.3\]

Where:
- $\varepsilon_{c\sigma}(t)$ – is the stress-dependent strain in concrete at time (t):
  $$\varepsilon_{c\sigma}(t) = \varepsilon_{ci}(t_0) + \varepsilon_{cc}(t) \ [15]$$
  \[3.4\]
- $\varepsilon_{cn}(t)$ – is the stress-independent strain in concrete at time (t):
  $$\varepsilon_{cn}(t) = \varepsilon_{cs}(t) + \varepsilon_{cT}(t) \ [14]$$

Although not strictly correct, it is usually assumed that all four components are independent, can be calculated separately and can be combined to obtain the total strain [16].

Immediately after the concrete sets or at the end of moist curing ($t=t_d$), shrinkage strains begin to develop and continue to increase at a decreasing rate. At the moment of the application of the stress, or at the moment of loading, ($t=t_0$), a sudden jump in the strain occurs which is called instantaneous or elastic strain. Under the effect of this applied sustained stress, the strain increases with time due to the creep of concrete.

3.2 Instantaneous (Elastic) Strain
The strain which occurs during application of stress is referred to as elastic, initial or instantaneous strain. For service loads, it is usually assumed that the stress in concrete is proportional to strain, and therefore the instantaneous strain can be expressed as follows:

$$\varepsilon_{c\sigma}(t_0) = \frac{\sigma_c(t_0)}{E_c(t_0)} \ [15]$$

\[3.4\]

Where $\sigma_c(t_0)$ is the applied concrete stress at time $t_0$, while $E_c(t_0)$ is the secant modulus of elasticity at the same age when the stress is applied and is defined in Figure 3.2.
3.3 Creep

When concrete is subjected to a sustained stress, an instantaneous strain occurs and gradually increases with time due to the creep of concrete. In the period immediately after the initial loading, creep develops rapidly and the rate of increase is slowed down with time. It is considered that about 50% of the final creep develops in the first 2-3 months and about 90% develops after 2-3 years. Due to the fact that creep and shrinkage never occur as separate phenomena, creep is usually calculated as the difference between the total time-dependent strain of a loaded specimen and the shrinkage of the same, but unloaded specimen.

When the sustained concrete stress is less than 0.5f′c, the creep is almost proportional to the stress and it is called a linear creep. If the stress levels are higher, which is almost never the case...
in service, the creep increases at a faster rate due to the increased micro-cracking and becomes non-linear with respect to stress.

Generally, the creep is composed of basic creep, which is a time-dependent strain in a loaded specimen, which is in hygral equilibrium with the ambient medium (no drying occurs), and drying creep, which is the additional creep that occurs in a drying specimen.

Creep is caused by many complex mechanisms that are still not fully understood. It includes many micro-mechanisms which originate from the hardened cement paste that consists of a solid cement gel, which contains many capillary pores and is made of colloidal sheets of calcium silicate hydrates separated by spaces containing absorbed water. The difference in the amount of the capillary pores between normal and high-strength concrete actually causes bigger creep strains in normal concrete. Neville et al. identified the following mechanisms for creep:

- Sliding of the colloidal sheets in the cement gel between the layers of absorbed water – viscous flow;
- Expulsion and decomposition of the interlayer water within the cement gel – seepage;
- Elastic deformation of the aggregate and the gel cristals as viscous flow and seepage occur within the cement gel – delayed elasticity;
- Local fracture within the cement gel involving the breakdown (and formation) of physical bonds – microcracking;
- Mechanical deformation theory and
- Plastic flow.

The magnitude and rate of development of creep depend on many factors. Creep increases when:

- the concrete strength decreases;
- the content of the aggregate decreases;
- less stiffer aggregates are used;
- the water/cement ratio increases;
- the relative humidity decreases;
- there is an increase in temperature which accelerates drying;
- the intensity of the sustained stress is increased;
- the duration of the stress is longer, and
- the age of the concrete when the stress was first applied is smaller.

In the following figure, the effect of the age at first loading on the creep-time curves of identical specimens first loaded at ages $\tau_0$, $\tau_1$, and $\tau_2$, is presented.
Due to its complexity, it is better to subdivide creep into few characteristic creep strain components.

When the load is removed, instantaneous creep recovery appears, followed by a delayed elastic creep recovery. The delayed elasticity is thought to be caused by the elastic aggregate acting on the viscous cement paste after the applied load is removed. If the specimen is unloaded after a long period under load, the magnitude of the delayed elastic creep strain is in the order of 40-50% of the instantaneous strain or 10-20% of the total creep strain. The larger part of the creep strain is irrecoverable and it is known as a flow creep. The flow creep can be subdivided into rapid initial flow, which occurs in the first 24 hours after loading and the remaining flow which occurs after the first day. The rapid initial flow depends mainly on the age at first loading, while the remaining flow develops gradually with time and can be divided into basic and drying flow. The basic flow depends on the composition of the concrete mix and the age at first loading, while the drying flow depends on the relative humidity, the gradient, the size and the shape of the specimen [16].

Creep is usually measured and introduced in the design of concrete structures in terms of the creep coefficient $\varphi(t, t_0)$. Creep coefficient at any time t is the ratio between the creep strain and the instantaneous strain:

$$\varphi(t, t_0) = \frac{\varepsilon_{cc}(t, t_0)}{\varepsilon_{ci}(t_0)}$$  \[16\]
Therefore, the creep strain at time $t$ caused by a constant sustained stress $\sigma_\alpha(t_0)$, first applied at age $t_0$, is:

$$\epsilon_\alpha(t, t_0) = \varphi(t, t_0) \epsilon_\alpha(t_0) = \varphi(t, t_0) \frac{\sigma_\alpha(t_0)}{E_\epsilon(t_0)} \quad \text{(3.6)}$$

This way, if the creep coefficient is known, the creep strain can be calculated from any constant sustained stress at any time. It is assumed that, as time approaches infinity, the creep coefficient reaches a final value that is usually in the range from 1.5 - 4.0.

The stress induced strain, meaning the instantaneous plus the creep strain caused by a constant sustained stress can be expressed by the following equation:

$$\epsilon_\alpha(t_0) + \epsilon_\alpha(t, t_0) = \frac{\sigma_\alpha(t_0)}{E_\epsilon(t_0)} + \varphi(t, t_0) \frac{\sigma_\alpha(t_0)}{E_\epsilon(t_0)} \left[1 + \varphi(t, t_0)\right] \quad \text{(3.7)}$$

### 3.4 Shrinkage

Shrinkage of concrete is a combination of several types of shrinkage:

- Plastic;
- Autogenous;
- Drying;
- Thermal and
- Carbonic shrinkage.

Plastic shrinkage occurs in wet concrete before setting, while all other types of shrinkage occur in hardened concrete after setting [16].

Autogenous shrinkage occurs due to hydration of the cement in a sealed specimen with no moisture exchange. It occurs rapidly in the days and weeks after casting and is less dependent on the environment and the size of the specimen than the drying shrinkage [16]. It has greater values for high-strength, self-compacting and massive concrete than those of normal strength concretes.

The most important type of shrinkage for normal strength concretes is the drying shrinkage, Figure 3.5, which occurs because of the movement of the water through the hardened concrete, i.e. evaporation of the internal water into the external environment. It starts after curing of concrete is finished. Drying shrinkage is smaller in high-strength concretes due to the smaller quantities of free water after hydration. The magnitude and rate of development of drying shrinkage depends on many factors: type and quantity of cement, type and quantity of any chemical admixtures and mineral additives, water content, water/cement ratio, type of aggregate, fine/course aggregate ratio, size and shape of specimen, curing regime and relative humidity and its change rate. Drying shrinkage increases when:

- the water/cement ratio increases;
- the content of the aggregate decreases;
- less stiffer aggregates are used;
- the relative humidity decreases;
- there is an increase in temperature which accelerates drying;
- fly ash or silica fume are used, and
the exposed surface area to volume ratio increases.

Figure 3.5: Drying shrinkage

Thermal shrinkage is the contraction that occurs in the first few hours or days after setting, as the heat of hydration gradually dissipates [16].

Carbonic shrinkage occurs near the surface of the concrete where CO₂ can react with Ca(OH)₂ to form CaCO₃. The carbonation process is accompanied by an increase in concrete weight and strength as well as reduced permeability and shrinkage. The carbonic shrinkage is not of much significance due to its small magnitude when compared to drying shrinkage, but if carbonation depth reaches the steel reinforcement, the steel becomes liable to corrosion.

3.5 Thermal Strain

The thermal strain produced by a change in temperature ΔT=T-T₀ can be calculated as:

$$\varepsilon_{ct}(t) = \int_{T_0}^{T} \alpha dT \ [16]$$

(3.8)

Where T₀ is the initial temperature and α is the coefficient of thermal expansión, which depends on the temperature and moisture content and is usually taken as $10 \times 10^{-6} \ ^\circ\text{C}$ [16].
CHAPTER 4: LONG-TERM BEHAVIOUR OF SFRC

4.1 Creep and Shrinkage of Steel Fibre Reinforced Concrete

4.1.1 General

The REPORT ON FRC published by ACI [24] shows that, according to limited test data on creep and shrinkage of SFRC, if fibres are used to the amount of less than 1% of the volume, there is no significant improvement in creep and shrinkage strain. Edgington et al. have reported that shrinkage of concrete over a period of three months is unaffected by the presence of steel fibres [11].

Balaguru and Ramakrishnan found that 0.5% of steel fibres slightly increase the creep of concrete and lead to less shrinkage strains [3]. Houde et al. found that 1.0% of steel fibres increase the creep strain by 20-40%. On the other hand, Chern and Chang found that steel fibres reduce the creep strain [5]. Swamy and Theodorakopoulos have reported that inclusion of 1% fiber results in improved creep properties of concrete under flexure [29]. Swamy and Stavrides have reported that drying shrinkage is reduced by about 15-20% (Figure 4.1) due to the addition of 1% fibres [27].

Hannant have reported that steel fibres have no significant effects on both creep and shrinkage properties of concrete [28].

Malmberg and Skarendahl, have reported that Steel fiber concrete with a fiber content of up to 2% undergoes less shrinkage than plain concrete [29].

Similar conclusion was reported by Young and Chern. They found out that the optimal volume fraction of steel fibres to reduce shrinkage is not more than 2%. Another conclusion from their research is that the larger aspect ratio of fibres leads to smaller shrinkage strains (Figure 4.2). They also proposed a modification of the BP model for calculation of the shrinkage of SFRC. The parameters that they included in the modification were the volume fraction and the aspect ratio of the steel fibres [33].

Figure 4.1: Influence of fiber addition on the free drying shrinkage of concrete [27]
4.1.2 A theory of Creep of Steel Fibre Reinforced Cement Matrices under Compression

The creep of concrete under uniaxial compression consists of two main components: instantaneous elastic strain and flowing creep. The instantaneous elastic component is formed immediately after the application of the load, while the flowing creep is very small in the beginning and increases with time. The addition of steel fibres does not significantly affect the instantaneous elastic strain, but they provide restraint to the sliding action of the matrix relative to the fibres which occurs through the fibre-matrix interfacial bond strength. The theoretical model was validated with experimental data at stress-strength ratios of 0.3 and 0.55 and, finally, an empirical expression was derived to determine the creep of steel fibre reinforced concrete based on the knowledge of fibre size, volume fraction, coefficient of friction and creep in ordinary concrete.

The idealized distribution of randomly oriented steel fibres in the direction of the applied stress is presented in Fig.4.3. Each fibre is considered to be surrounded by a thick cylinder of cement matrix with diameter \( s \) equal to the spacing between fibres and length \( l_e/2+s/2 \), where \( l_e \) is equivalent length in the direction of the stress equal to 0.41 from the total length of the fibre \( l \). The interfacial bond stress \( \tau \) is activated to resist the flow component of creep.

Steel fibres become more effective in restraining creep as the age under load increases. This is due to the fact that they affect only the flow component, which is bigger at a later age, while the instantaneous and delayed elastic components are dominant at an earlier age.
The authors, Mangat and Azari, proposed a theoretical expression to predict the creep strain of SFRC, \( \varepsilon_{fc} \), at a stress-strength ratio of 0.3, based on knowledge of the creep strain of ordinary concrete \( \varepsilon_{oc} \), coefficient of friction \( \mu \), fibre volume \( \nu_f \) and aspect ratio of the fibres \( l/d \):

\[
\varepsilon_{fc} = \varepsilon_{oc} \left(1 - 1.96\mu \nu_f \frac{l}{d} \right) \quad [18]
\] (4.1)

The only unknown parameter in this equation is the coefficient of friction \( \mu \), which was obtained by using the experimental data of the authors and also data from other sources. The coefficient of friction was considered a material property of the interface governed by the surface texture, the shape of the fibers and the mix proportions of the concrete. The average values of \( \mu \) for normal concrete mixes and hooked end steel fibres range between 0.08 -0.12, for crimped steel fibres being 0.12, while for straight steel fibres being 0.04.

It is important to mention that steel fibres are more effective in restraining the creep at lower sustained stress-strength ratios because of the smaller lateral deformation caused by the sustained axial stress. This results in greater values of \( \tau \) and therefore greater fibre restraint to creep.

According to this expression, the decreasing of creep of SFRC, compared to plain concrete, ranges from 0 to 30\%.
4.1.3 A Theory of Free Shrinkage of Steel Fibre Reinforced Cement Matrices [19]

The restraint provided by aggregate particles to the shrinkage of concrete is well recognized. The mechanism of restraint is realized through idealized spherical aggregate inclusions which restrain deformations in a radial direction. On the other hand, addition of fibres in concrete provide additional restraint as presented in Fig.4.3. However, the mechanism of restraint is different in this case since the contact area at the tip of the fibre is too small to allow any restraint to matrix deformation parallel to the fibre. The shrinkage matrix has a tendency to slide past the length of the fibre and restraint is only possible through the fibre-matrix interfacial bond strength. Mangat and Azari proposed a theoretical expression to predict shrinkage of SFRC, based on the knowledge of shrinkage of ordinary concrete \( \epsilon_{os} \), coefficient of friction \( \mu \), fibre volume \( \nu_f \) and aspect ratio of the fibres \( l/d \):

\[
\epsilon_{fs} = \epsilon_{os} \left( 1 - 2.45 \mu \nu_f \frac{l}{d} \right) \quad [19]
\]

According to this expression, the decreasing of shrinkage of SFRC, compared to plain concrete, ranges from 0 to 40%.

4.2 Time-Dependent Structural Response of SFRC Beams

4.2.1 General

Many analytical and experimental studies have been conducted for prediction of short-term deflection of SFRC beams. Swamy and Al-Noori have reported that steel fibres aid in deflection control. Swamy, Al-Ta'an and Ali concluded that the presence of steel fibres led to an increase in the stiffness of the beams, which resulted in reduction of the deflections. Kormeling, Reinhardt and Shah tested beams reinforced with longitudinal reinforcement and steel fibres and they also concluded that the addition of fibers led to a smaller beam deflection [29]. An analytical model for calculation of deflections has been proposed by Lim, Paramasivam and Lee in which the load-deflection relationship of simply supported SFRC beams is derived by use of an idealized stress-strain relationship of SFRC in tension and compression. Craig and Craig et al. proposed a model in which the moment-rotation and moment-curvature diagrams are computed by using experimentally derived stress-strain relationship [28]. Soroushian and Reklaoui proposed a similar method [29], while Alsayed proposed an experimental model that accounts for the volume fraction and aspect ratio of the fibers [28].

Tan, Paramasivam and Tan found that steel fibers enhanced the first-crack flexural strength and improved the flexural stiffness (Fig.4.4) and therefore reduced the deflections. Load-deflection curves from the short-term testing to failure are presented in Figure 4.5. It can be noticed that the higher the fiber content, the higher is the load at first cracking. When the load was increased beyond the cracking load, there was an increase in the beam stiffness with increasing of the fiber content. Fibers also increased the load at which yielding of the tensile reinforcement starts. The instantaneous deflection was reduced for 30% with addition of 2% steel fibres when compared to the concrete without fibres.
Figure 4.4: Variation of effective flexural stiffness (EI) with applied moment [28]

Figure 4.5: Load-deflection curves of SFRC beams [28]

On the other hand, research studies dealing with long-term behavior of SFRC beams are very rare. The long term behavior of SFRC beams under sustained load has been investigated only by Tan, Paramasivam and Tan in 1994 as a one year study and later the same research was continued as a ten year study of the deflections and crack widths under sustained load by Tan and Saha in 2005. Another research [32] published by Vasanelli, Micelli, Aiello and Plizzari in 2012 is dealing with the influence of long term sustained loading on the cracking behavior and consequently on the structural durability.

4.2.2 Long-Term Deflections of SFRC Beams

The experimental program of Tan, Paramasivam and Tan consists of 14 SFRC beams subjected to various sustained load levels and reinforced with different amount of steel fibres. Five beams were tested to failure under third-point loading at the age of 28 days, while nine beams were subjected to long-term flexural creep tests for about one year. The beams were 200cm long, simply supported with clear span length of 180cm. The cross section of the beams was b/d=10/12.5cm, reinforced with 2Ø10 in the tensile and 2Ø6 in the compressive zone. As a
shear reinforcement, stirrups ø6/7.5cm, were used. The test setup is presented in Figure 4.6.

![Figure 4.6: Sustained load test setup [29](image)](image)

The experimental program of the beams under long-term loading and beam designation are presented in the subsequent table.

**Table 4.1: Experimental program [28]**

<table>
<thead>
<tr>
<th>Beam designation</th>
<th>Steel fiber content, %</th>
<th>Sustained load level (as a ratio to design ultimate load)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A - 50</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>B - 50</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>C - 50</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>D - 50</td>
<td>1.5</td>
<td>0.5</td>
</tr>
<tr>
<td>E - 50</td>
<td>2.0</td>
<td>0.5</td>
</tr>
<tr>
<td>C - 0</td>
<td>1.0</td>
<td>Self-weight</td>
</tr>
<tr>
<td>C - 35</td>
<td>1.0</td>
<td>0.35</td>
</tr>
<tr>
<td>C - 65</td>
<td>1.0</td>
<td>0.65</td>
</tr>
<tr>
<td>C - 80</td>
<td>1.0</td>
<td>0.8</td>
</tr>
</tbody>
</table>

The results for the beams subjected to sustained loads equal to 0.5 times the design strength in flexure, are shown in Figure 4.7. It can be concluded that long-term deflections decrease with the increase of the fiber dosage. 2% steel fibres reduced the long-term deflections for about 20%.

Another conclusion from this research is that steel fibres are more effective in reducing long-term deflections at load levels higher than 0.5 times the design ultimate flexural strength.

Finally, an empirical expression for prediction of long-term deflections of SFRC beams was proposed, following the ACI approach, which gives good results for sustained load levels of up to 0.5 times the design ultimate flexural strength. The ACI Building code uses an empirical formula by which the additional long-term deflection $\Delta_s$ due to sustained load is obtained by multiplying the initial elastic deflection $\Delta_i$ by a factor $\lambda$. $\lambda$ depends on the compression steel ratio $\rho'$ and the material modifier depending on the creep and shrinkage $\xi$, usually in the range of 0-2. For long-term sustained load with a duration of one year, $\xi = 1.4$.

$$\Delta_s = \lambda \Delta_i \quad [28]$$

$$\lambda = \frac{\xi}{1 + 50 \rho'} \quad [28]$$
They propose introduction of a factor $\alpha$ for SFRC which depends on the aspect ratio of the fiber $l / d_f$, volume fraction of fibers $V_f$, coefficient of friction $\mu$ (0.04 for straight and 0.12 for deformed fibers) and experimental constant $c$, which was determined from the linear regression analysis to be 0.96.

$$\Delta_i = \alpha \lambda \Delta_i = \left[1 - c \mu \frac{l}{d_f} V_f\right] \lambda \Delta_i \quad [28] \quad (4.5)$$

From the performed regression analysis, they found out that, for a volume fraction of up to 1.5%, $\alpha = 1 - 0.069 V_f$, while for a volume fraction of more than 1.5%, $\alpha = 0.897$.

They also proposed certain modifications of the Effective Modulus Method (EMM) and the Adjusted Effective Modulus Method (AEMM).

![Figure 4.7: Long-term deflection of SFRC beams [28]](image)

The same beams that were used by Tan, Paramasivam and Tan were later subjected to sustained load for ten years by Tan and Saha. The geometry and test setup were the same as presented in Figure 4.6. The experimental program was slightly changed and is presented in Table 4.2. The beams were divided into two series. Series I were beams with steel fiber content varying from 0 to 2%. All of these were subjected to a sustained load equal to 50% of the ultimate load of the beam without fibers. Series II contained beams with the same steel fiber content of 1%, but all beams were subjected to various load levels, from 0.35 – 0.80 times the ultimate load. At the end of the experiment, the sustained load was removed from all beams and they were reloaded to failure.
Table 4.2: Experimental program [29]

<table>
<thead>
<tr>
<th>Series</th>
<th>Beam designation</th>
<th>Steel fiber content, %</th>
<th>Sustained load level (as a ratio to design ultimate load)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>A - 50</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>B - 50</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>C - 50</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>D - 50</td>
<td>1.5</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>E - 50</td>
<td>2.0</td>
<td>0.5</td>
</tr>
<tr>
<td>II</td>
<td>C - 35</td>
<td>1.0</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>C - 50</td>
<td>1.0</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>C - 59</td>
<td>1.0</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>C - 65</td>
<td>1.0</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>C - 80</td>
<td>1.0</td>
<td>0.80</td>
</tr>
</tbody>
</table>

The results presented in Figure 4.8 (a and b) for the Series I, indicate that the total and the long-term deflection were more reduced with the increase of the fiber content. After a year, the deflection of the beam without fibers (A-50) exceeded that of the beam with 2% steel fibers (E-50) for 29%. After ten years, this difference was 36%. If only the long-term deflection is considered, the difference is 41% and 51%. The long-term/instantaneous deflection ratio (Fig.4.8(c)) did not differ much after a year, while after ten years, it was 1.03 for beam A-50 and 1.30 for beam E-50. As the period of sustained load is increased, the fibers become more effective in decreasing the deflections.

Figure 4.8: SFRC beams (P_s/P_u=0.5) with various fiber contents [29]
The influence of the sustained load level on beams reinforced with the same amount of fibers (1%) is presented in Figure 4.9. The total and long-term deflection increased with the increase of the sustained load level, while the long-term/instantaneous deflection ratio decreased with the increase of the sustained load level.

4.2.3 Long-Term Crack Widths of SFRC Beams

Based on the results from their research, Tan, Paramasivam and Tan proposed an analytical expression for short-term crack width of a SFRC beam $w_f$, related linearly to a RC beam without fibers $w_i$.

$$w_f = (1 - k_i V_f) w_i$$ [28] ...........................................................................................................(4.6)

Where:

- $V_f$ is the volume fraction of fibers;
- $k_i = 0.22$, an experimental constant to account for the fiber properties.

They have observed that the ratio of long-term to short-term crack width decreases with a fiber content of up to 0.5%, and increases with a fiber content of more than 0.5%.

The proposed equations are as follows:

- for $V_f < 0.5$

$$\frac{w_f}{w_{ir}} = \frac{c_1}{c_2 + V_f} + c_3 + c_4 V_f$$ [28] ...........................................................................................................(4.7)
for $V_f > 0.5$

$$\frac{w_{lt}}{w_{st}} = k_2 + k_3V_f \quad [28]$$ ................................................................. (4.8)

Where:

- $V_f$ is the volume fraction of fibers;
- $w_{lt}$ is the long-term crack width;
- $w_{st}$ is the short-term crack width;
- $c_1, c_2, c_3, k_2, k_3$ are experimentally derived constants.

Later, after a ten year research, Tan and Saha performed nonlinear regression analysis and found out that $c_1 = 0.03, c_2 = 0.04, c_3 = 0.253, c_4 = 0.12$. By linear regression analysis, they also found out that $k_2 = 0.276, k_3 = 0.12$. They used the previous equations and found out a good correlation with the experimental results.

As mentioned earlier, in the ten year research of Tan and Saha, the crack width development under the effect of sustained loads was also measured. The subsequent figure shows the crack widths over the period of ten years. It can be noticed that the addition of fibers decreases the crack width. The stabilization of the crack width development occurs earlier in beams with higher fiber content (Fig.4.9a) and in beams subjected to a lower sustained load level (Fig.4.9b).

In the case of the beams with the highest content of fibers (E-50 and D-50), there was almost no increase (7% for E-50) of the crack width after a year, while in the case of the beam with no fibers (A-50), the increase was 24% in the course of one to ten years.

The beam under the lowest sustained load level (C-35) exhibited a constant maximum crack width throughout the whole research.

![Figure 4.9: Maximum crack widths for beams with various fiber content (Series I) and various sustained load levels (Series II) [29]](image)
CHAPTER 5: STRESS-STRAIN RELATIONS FOR SFRC AND REINFORCEMENT

5.1 Stress-Strain Relation of SFRC under Short-Term Load

5.1.1 Stress-Strain Relation of SFRC under Compression

Stress-strain curves for concrete with different steel fibre contents during uniaxial compression have been obtained by many authors. In general, the incorporation of the usual amounts of fibers in practice does not significantly affect the compressive strength of concrete. Therefore, design recommendations such as those of DBV (Deutche Beton Verein) and RILEM, consider compressive strength of SFRC to be the same as that of plain concrete. RILEM emphasizes that EC2 has been used as the basis for their proposal [25] wherefore the stress-strain relation from EC2 is presented in Figure 5.1.

\[
\frac{\sigma_c}{f_{cm}} = \frac{k \eta - \eta^2}{1 + (k - 2) \eta} \quad [13]
\]

Figure 5.1: Stress-strain relation for concrete and SFRC under compression [13]

\[
\frac{\sigma_c}{f_{cm}} = \frac{k \eta - \eta^2}{1 + (k - 2) \eta} \quad [13]
\]

Where:

- \( \eta = \varepsilon_c / \varepsilon_{ct} \)
- \( \varepsilon_{ct} \) is strain at maximum stress
- \( k = 1.05 \ E_{cm} \times |\varepsilon_{ct}|/f_{cm} \)

Expression (5.1) is valid in case of \( 0 < |\varepsilon_c| < |\varepsilon_{cur}| \) where \( \varepsilon_{cur} \) is the nominal ultimate strain [13].

For design of cross-sections, the following relationship between stresses and strains shown in figure 5.2 and given with equations (5.2) and (5.3), can be used [13]:

\[
\sigma_c = f_{cd} \left[ 1 - \left( 1 - \frac{\varepsilon_c}{\varepsilon_{c1}} \right)^n \right] \quad \text{for} \quad 0 \leq \varepsilon_c \leq \varepsilon_{c2} \quad [13]
\]

\[
\sigma_c = f_{cd} \quad \text{for} \quad \varepsilon_{c1} \leq \varepsilon_c \leq \varepsilon_{c2} \quad [13]
\]

where:

- \( f_{cd} \) is the design compressive strength
\( N \) is an exponent;
\( \varepsilon_{c2} \) is the strain at maximum stress;
\( \varepsilon_{cu2} \) is the ultimate strain.

![Parabola-rectangle diagram for concrete under compression](image)

Figure 5.2: Parabola-rectangle diagram for concrete under compression [13]

Other simplified stress-strain relationships may be used, like the bilinear relation [13].

SFRC is classified in respect to its compressive strength by SFRC strength classes, which are related to the cylinder strength \( f_{\text{ck}} \) or the cube strength \( f_{\text{ck,cube}} \). The strength classes are the same as those for plain concrete and are presented in Table 5.1 [25].

<table>
<thead>
<tr>
<th>Strength class of SFRC</th>
<th>C20/25</th>
<th>C25/30</th>
<th>C30/37</th>
<th>C35/45</th>
<th>C40/50</th>
<th>C45/55</th>
<th>C50/60</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{\text{ck}} ) (MPa)</td>
<td>20</td>
<td>25</td>
<td>30</td>
<td>35</td>
<td>40</td>
<td>45</td>
<td>50</td>
</tr>
<tr>
<td>( f_{\text{ck,cube}} ) (MPa)</td>
<td>25</td>
<td>30</td>
<td>37</td>
<td>45</td>
<td>50</td>
<td>55</td>
<td>60</td>
</tr>
<tr>
<td>( f_{\text{ctm,fl}} ) (MPa)</td>
<td>3.7</td>
<td>4.3</td>
<td>4.8</td>
<td>5.3</td>
<td>5.8</td>
<td>6.3</td>
<td>6.8</td>
</tr>
<tr>
<td>( f_{\text{ctk,fl}} ) (MPa)</td>
<td>2.6</td>
<td>3.0</td>
<td>3.4</td>
<td>3.7</td>
<td>4.1</td>
<td>4.4</td>
<td>4.8</td>
</tr>
<tr>
<td>( E_{\text{fcm}} ) (GPa)</td>
<td>29</td>
<td>30.5</td>
<td>32</td>
<td>33.5</td>
<td>35</td>
<td>36</td>
<td>37</td>
</tr>
</tbody>
</table>

As it can be noticed from Table 5.1, strength classes only up to C50/60 were proposed by RILEM [25]. Steel fibres can also be used in high strength concrete, but care should be taken that steel fibres do not break in a brittle way, before being pulled out [25].

5.1.2 Stress-Strain Relation of SFRC under Tension

The stress-strain relation of SFRC when subjected to tensile loads is much more different than that of plain concrete. Different testing methods can be used to obtain the tensile strength of SFRC: uniaxial tensile tests, 3 or 4 point bending tests and splitting or wedge splitting tests. Due to the complexity of the uniaxial tests, bending tests are mainly used to obtain the behavior of SFRC under tension.
In 2001, DVB-Merkblatt Stahlfaserbeton proposed a trilinear stress-strain relation for SLS and a bilinear one for ULS. It is based on 4 point bending tests and includes equivalent flexural tensile strengths and size factor to characterize the post-cracking behavior. The proposal for a stress-strain relation given by RILEM TC162-TDF in 2000, known as the original RILEM stress-strain relation, also used the trilinear form of the stress-strain relation and equivalent flexural tensile strengths, but based on a 3 point bending test, without the size factor. After many performed tests, it was concluded that the original RILEM stress-strain relation can be improved. The updated RILEM stress-strain relation (Figure 5.3) was published in 2003, [25] and, instead of equivalent flexural tensile strengths, it uses residual flexural tensile strengths and a size factor to characterize the post-cracking behavior of SFRC. It was found that residual strengths provide more certainty about the material capacity at a certain crack width. Other important modification is the change of the maximum strain from 10 to 25‰ [10]. Further on, the updated RILEM stress-strain relation is presented in Figure 5.3 and is explained in details.

If only compressive strength $f_{ck}$ is determined, the estimated mean and characteristic flexural tensile strength of SFRC will be derived from the following equations, taking into account the values given in Table 5.1:

$$f_{ctm,ax} = 0.3 \cdot (f_{ck})^{\frac{1}{3}} (N/mm^2) \ [25]$$ .......................................................... (5.4)

$$f_{ctk,ax} = 0.7 \cdot f_{ctm,ax} (N/mm^2) \ [25]$$ .......................................................... (5.5)

$$f_{ctl,ax} = 0.6 \cdot f_{ctl,lt} (N/mm^2) \ [25]$$ .......................................................... (5.6)

$$f_{ctk,lt} = 0.7 \cdot f_{ctm,lt} (N/mm^2) \ [25]$$ .......................................................... (5.7)

If bending tests are performed, the characteristic value of the proportionality limit (LOP) can be calculated as:

$$f_{ctk,l} = f_{ctm,l} - k_s s_p (N/mm^2) [25]$$ .......................................................... (5.8)

Where:

- $f_{ctk,l}$ is the characteristic value of LOP ($N/mm^2$);
- $f_{ctm,l}$ is the mean value of LOP ($N/mm^2$);
- $s_p$ is the standard deviation ($N/mm^2$);
- $k_s$ is a factor depending on the number of specimens (Table 5.2).

The maximum value of equations 5.7 and 5.8 can be taken as the flexural tensile strength of SFRC.
The load at the proportionality limit $F_L$ is equal to the highest value of the load in the interval $\delta$ of 0.05mm.

More important than the flexural strength in SFRC are the residual flexural tensile strengths. They can be determined either by crack mouth opening displacement (CMOD) or by deflection controlled bending test. The bending test is performed on standard notched test specimens with cross section 150/150mm, with a minimum length of 550mm and span of 500mm. The width of the notch is not larger than 5mm and the beam has an unnotched depth of 125mm. The two supports and the device for imposing of the displacement are steel rollers with a diameter of 30mm. The testing machine must have big stiffness to avoid unstable zones in the $F-\delta$ curve and should be operated so that the measured deflection of the specimen at mid span increases at a constant rate of 0.2mm/min. During the whole testing, the load and the mid span deflection must be recorded continuously [26].

**Table 5.2: Values of factor $k_x$ [25]**

<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_x, \text{known}$</td>
<td>2.31</td>
<td>2.01</td>
<td>1.89</td>
<td>1.83</td>
<td>1.80</td>
<td>1.77</td>
<td>1.74</td>
<td>1.72</td>
<td>1.68</td>
<td>1.67</td>
<td>1.64</td>
</tr>
<tr>
<td>$k_x, \text{unknown}$</td>
<td>-</td>
<td>-</td>
<td>3.37</td>
<td>2.63</td>
<td>2.33</td>
<td>2.18</td>
<td>2.00</td>
<td>1.92</td>
<td>1.76</td>
<td>1.73</td>
<td>1.64</td>
</tr>
</tbody>
</table>

**Figure 5.3: Stress-strain diagram and size factor [25]**
The residual flexural tensile strengths, \( f_{R,i} \) and \( f_{R,4} \), are defined at the following crack mouth opening displacements (CMODi) or mid-span deflections (\( \delta_{R,i} \)): 
- \( f_{R,1} \) at CMOD1=0.5mm or \( \delta_{R,1}=0.46\)mm;
- \( f_{R,4} \) at CMOD4=3.5mm or \( \delta_{R,4}=3.00\)mm.

\( f_{R,i} \) and \( f_{R,4} \) can be calculated with the following equation:

\[
 f_{R,i} = \frac{3 F_{R,i} L}{2 b h^2_{sp}} \quad (N/m^2) \quad [25] 
\]

Where:
- \( b \) is the width of the specimen (mm);
- \( h_{sp} \) is the distance between the tip of the notch and the top of the cross section (mm);
- \( L \) is the span of the specimen (mm);
- \( F_{R,i} \) is the force recorded at the previously stated CMODi or \( \delta_{R,i} \) (N).

The classification of SFRC can be done by its residual strength class FL, followed by two parameters that are determined using the residual flexural tensile strengths, \( f_{R,1} \) and \( f_{R,4} \). The first parameter, FL0.5, is given by the value of \( f_{R,1} \) reduced to the nearest multiple of 0.5 MPa, while the second parameter, FL3.5, is given by the value of \( f_{R,4} \) reduced also to the nearest multiple of 0.5 MPa. The first parameter can vary between 1 and 6 MPa and the second between 0 and 4 MPa. They denote the minimum guaranteed characteristic residual strengths at CMOD values of 0.5 and 3.5mm. In this way, the residual strength class follows the strength class of SFRC when it needs to be classified, for example C30/37 FL 2.0/1.5 [25].

### 5.2 Stress-Strain Relation of Concrete under Long-Term Load

#### 5.2.1 Creep Coefficient, Specific Creep and Compliance Function [16]

As previously stated in Chapter 3.3, creep coefficient at any time \( t \) is the ratio between the creep strain and the instantenous strain:

\[
 \phi(t, t_0) = \frac{\varepsilon_c(t, t_0)}{\varepsilon_c(t_0)} \quad [16] \]

Therefore, the creep strain at time \( t \) caused by a constant sustained stress \( \sigma_c(t_0) \), first applied at age \( t_0 \), is:

\[
 \varepsilon_c(t, t_0) = \phi(t, t_0) \varepsilon_c(t_0) = \phi(t, t_0) \frac{\sigma_c(t_0)}{E_c(t_0)} \quad [16] \]

Another frequently used time function is the specific creep, \( C(t, t_0) \), which is a proportionality factor relating stress to linear creep:

\[
 \varepsilon_c(t, t_0) = C(t, t_0) \sigma_c(t_0) \quad \text{or} \quad C(t, t_0) = \frac{\varepsilon_c(t, t_0)}{\sigma_c(t_0)} \quad [16] \]

The specific creep, \( C(t, t_0) \), is the creep strain at time \( t \), produced by a unit stress, first applied at age \( t_0 \).
The relationship between the creep coefficient and specific creep can be obtained from the previous equations as:
\[
\varphi(t, t_0) = C(t, t_0)E_c(t_0) \quad [16] \quad \text{.................................................................................(5.14)}
\]

The sum of the instantaneous and creep strains at time \( t \), produced by a sustained unit stress applied at \( t_0 \), is defined as the creep function or compliance function, \( J(t, t_0) \) and can be expressed as:
\[
J(t, t_0) = \frac{1}{E_c(t_0)} + C(t, t_0) = \frac{1}{E_c(t_0)} \left[ 1 + \varphi(t, t_0) \right] \quad [16] \quad \text{.................................................................................(5.15)}
\]

In this way, the stress induced strain, which means instantaneous plus creep strain, caused by a constant sustained stress \( \sigma_c(t_0) \), first applied at age \( t_0 \), is:
\[
\varepsilon_c(t_0) + \varepsilon_\infty(t, t_0) = J(t, t_0)\sigma_c(t_0) = \frac{\sigma_c(t_0)}{E_c(t_0)} \left[ 1 + \varphi(t, t_0) \right] = \frac{\sigma_c(t_0)}{E_c(t, t_0)} \quad [16] \quad \text{.................................................................................(5.16)}
\]

Where \( E_c(t, t_0) \) is known as the effective modulus and can be expressed as:
\[
E_c(t, t_0) = \frac{E_c(t_0)}{1 + \varphi(t, t_0)} \quad [16] \quad \text{.................................................................................(5.17)}
\]

### 5.2.2 The Principle of Superposition

As mentioned earlier, when subjected to service stress range of up to 40 or 50% of the compressive strength, the load-dependent strains in concrete are linearly related to stress. Therefore, the principle of superposition can be used to calculate the deformation caused by a time-varying stress history. The principle of superposition (Fig. 5.4) was first proposed for non-aging phenomena by Boltzmann (1874) and for aging phenomena by Volterra (1909) [17] and was first applied to concrete by McHenry, who stated that the strain produced by a stress increment applied at any time \( \tau \), is not affected by any stress applied either earlier or later.

This principle agrees well with experimental observations for increasing stress histories (Fig. 5.4c) where the creep curve is assumed to be equal to the sum of the creep curves produced by each stress increment acting separately. On the other hand, for decreasing stress histories (Fig. 5.4d), the principle of superposition overestimates creep recovery. The time at which the load is applied in this section is denoted as \( \tau \).

In Figure 5.5a, a stress history consisting of two stress increments \( \Delta \sigma_c(\tau_0) \) and \( \Delta \sigma_c(\tau_1) \), applied at times \( \tau_0 \) and \( \tau_1 \), is shown. The creep strain in this case at any time \( t > \tau \), can be determined by two creep coefficients, which are shown in Figure 5.5b, while the creep compliance functions are given in Figure 5.5c.
If the principle of superposition is applied, the total stress-dependent strain in concrete at time $t$ can be expressed as:

$$\varepsilon_{\sigma}(t) + \varepsilon_{\infty}(t) = \frac{\Delta \sigma_{\varepsilon}(\tau_0)}{E_{\varepsilon}(\tau_0)} [1 + \varphi(t, \tau_0)] + \frac{\Delta \sigma_{\varepsilon}(\tau_1)}{E_{\varepsilon}(\tau_1)} [1 + \varphi(t, \tau_1)] = \frac{\Delta \sigma_{\varepsilon}(\tau)}{E_{\varepsilon}(\tau)} [1 + \varphi(t, \tau)] \quad [16] \ldots (5.18)$$

Or written in terms of creep functions:

\[ \varepsilon_{\sigma}(t) + \varepsilon_{\infty}(t) = \frac{\Delta \sigma_{\varepsilon}(\tau_0)}{E_{\varepsilon}(\tau_0)} [1 + \varphi(t, \tau_0)] + \frac{\Delta \sigma_{\varepsilon}(\tau_1)}{E_{\varepsilon}(\tau_1)} [1 + \varphi(t, \tau_1)] = \frac{\Delta \sigma_{\varepsilon}(\tau)}{E_{\varepsilon}(\tau)} [1 + \varphi(t, \tau)] \quad [16] \ldots (5.18) \]
If we consider a continuously varying stress history, as presented in Figure 5.6, by dividing the time under load into n time steps, the stress can be approximated by a series of small stress increments applied at the end of each time step, wherefor each small stress increment, different creep coefficient is required as an input. According to the principle of superposition, in this case, the total strain in concrete at time t can be obtained by summing the stress-produced strains and the shrinkage strain:

\[
\varepsilon(t) = \varepsilon_{ci}(t) + \varepsilon_{sc}(t) + \varepsilon_{sh}(t)
\]

\[
\varepsilon(t) = \sum_{i=0}^{n} J(t, \tau_i) \Delta \sigma_c(\tau_i) + \varepsilon_{sh}(t) \tag{5.19}
\]

If we consider infinitesimal stress increments \( d\sigma_c(\tau) \), the summation in the previous equation can be written in an integral form:

\[
\varepsilon(t) = \int_{t_0}^{t} J(t, \tau) d\sigma_c(\tau) + \varepsilon_{sh}(t) = J(t, t_0) \sigma_{c0}(t_0) + \int_{t_0}^{t} \frac{1}{E_c(\tau)} d\sigma_c(\tau) + \varepsilon_{sh}(t) \tag{5.20}
\]

This equation represents an integral-type creep law and the integral for the stress-produced strains is the area under the graph of creep function versus concrete stress (Fig. 5.6b).

![Figure 5.6: Time-varying stress history and stress-produced strain](image)

Figure 5.6: Time-varying stress history and stress-produced strain [16]

The method is also known as step by step method (SSM). The main idea is that the stress is assumed to remain constant during each of the small time intervals. The greater the number of time intervals, the more accurate is the final prediction. The time discretisation should be such that an approximately equal portion of the creep coefficient develops during each time step.

The only disadvantage of this method is that it involves a certain reduced accuracy when applied to a decreasing load history and that large amount of creep data is required to preform it.

5.3 Actual Models for Design of Creep and Shrinkage of Concrete

5.3.1 Fib Model Code 2010 [14]

5.3.1.1 Range of Applicability

This model for creep and shrinkage predicts the time-dependent mean cross-section behaviour of a concrete member, moist cured at normal temperatures for not longer than 14 days.
Unless special provisions are given, the model is valid for ordinary structural concrete
\(20 \, \text{MPa} \leq f_{cm} \leq 130 \, \text{MPa}\), subjected to a compressive stress \(\sigma_c \leq 0.4 \, f_{cm} (t_0)\) at an age at loading
\(t_0\) and exposed to a mean relative humidity in the range of 40 to 100 % at a mean temperature in
the range of 5 °C to 30 °C. The age at loading should be at least 1 day [14].

5.3.1.2 General

The total strain at time \(t\), \(\varepsilon_c(t)\), of a concrete member uniaxially loaded at time \(t_0\) with a constant
stress \(\sigma_c(t_0)\) may be expressed as follows:

\[
\varepsilon_c(t) = \varepsilon_c(t_0) + \varepsilon_{cc}(t) + \varepsilon_{cs}(t) + \varepsilon_{cT}(t) \quad [14]
\]

where:
\(\varepsilon_c(t_0)\) is the initial strain at loading;
\(\varepsilon_{cc}(t)\) is the creep strain at time \(t > t_0\);
\(\varepsilon_{cs}(t)\) is the shrinkage strain;
\(\varepsilon_{cT}(t)\) is the thermal strain.

The first two components represent the part of the strain which is stress dependent, while the
last two represent the stress independent strain.

The initial strain at loading is the most straightforward component and it depends on the applied
stress and the tangent modulus of elasticity at an age \(t_0\):

\[
\varepsilon_c(t_0) = \frac{\sigma_c(t_0)}{E_{ci}} \quad [14]
\]

5.3.1.3 Creep

In reality, creep is a non-linear phenomenon. Since concrete is considered as an aging linear
visco-elastic material, creep is assumed to be linearly related to stress within the range of service
stresses \(\sigma_c \leq 0.4 \, f_{cm} (t_0)\).

Therefore, creep can be described by means of the creep coefficient \(\varphi(t,t_0)\) :

\[
\varepsilon_{cc}(t,t_0) = \frac{\sigma_c(t_0)}{E_{ci}} \varphi(t,t_0) \quad [14]
\]

where:
\(\varphi(t,t_0)\) is the creep coefficient;
\(E_{ci}\) is the modulus of elasticity at the age of 28 days.

If the stress dependent part of the total strain is denoted as \(\varepsilon_{cc}(t,t_0) = \varepsilon_{cc}(t_0) + \varepsilon_{cs}(t)\), then

\[
\varepsilon_{cc}(t,t_0) = \frac{1}{E_{ci}(t_0)} \left[ \frac{\varphi(t,t_0)}{E_{ci}} \right] = \sigma_c(t_0) J(t,t_0) \quad [14]
\]

where:
\(J(t,t_0)\) is the creep function or creep compliance, representing the total stress dependent strain
per unit stress;
\(E_{ci}(t_0)\) is the modulus of elasticity at the time of loading \(t_0\).

The creep coefficient may be calculated from:

\[
\varphi(t,t_0) = \varphi_{cc}(t,t_0) + \varphi_{sc}(t,t_0) \quad [14]
\]
where:

\( \varphi_{bc}(t, t_0) \) is the basic creep coefficient;

\( \varphi_{dc}(t, t_0) \) is the drying creep coefficient;

\( t \) is the age of concrete in days at the moment considered;

\( t_0 \) is the age of concrete at loading in days adjusted according to eq.5.38.

The basic creep coefficient \( \varphi_{bc}(t, t_0) \) can be calculated with the following equation:

\[
\varphi_{bc}(t, t_0) = \beta_{bc}(f_{cm}) \cdot \beta_{bc}(t, t_0) \quad [14]
\]

with:

\[
\beta_{bc}(f_{cm}) = \frac{1.8}{(f_{cm})^{0.3}} \quad [14]
\]

\[
\beta_{bc}(t, t_0) = \ln \left( \frac{30}{t_{0,adj}} + 0.035 \right) \cdot \left( t - t_0 \right) + 1 \quad [14]
\]

where:

\( f_{cm} \) is the mean compressive strength at an age of 28 days in MPa.

The drying creep coefficient \( \varphi_{dc}(t, t_0) \) can be calculated with the following equation:

\[
\varphi_{dc}(t, t_0) = \beta_{dc}(f_{cm}) \cdot \beta(RH) \cdot \beta_{dc}(t, t_0) \quad [14]
\]

with:

\[
\beta_{dc}(f_{cm}) = \frac{412}{(f_{cm})^{4}} \quad [14]
\]

\[
\beta(RH) = \frac{1 - \frac{RH}{100}}{0.1 \cdot \frac{h}{100}} \quad [14]
\]

\[
\beta_{dc}(t_0) = \frac{1}{0.1 + t_{0,adj}^{0.2}} \quad [14]
\]

The development of drying creep with time is described by:

\[
\beta_{dc}(t, t_0) = \left( \frac{t - t_0}{\beta_n + (t - t_0)} \right)^{\gamma(t_0)} \quad [14]
\]

with:

\[
\gamma(t_0) = \frac{1}{2.3 + \frac{3.5}{t_{0,adj}}} \quad [14]
\]

\[
\beta_n = 1.5h + 250\alpha \leq 1500\alpha \quad [14]
\]

with

\[
\alpha_{cm} = \frac{35}{f_{cm}^{0.5}} \quad [14]
\]
where:

- \( f_{cm} \) is the mean compressive strength at an age of 28 days in MPa;
- \( RH \) is the relative humidity of the ambient environment in %;
- \( h = 2Ac/u = \) notional size of member [mm], where \( Ac \) is the cross section [mm\(^2\)] and \( u \) is the perimeter of the member in contact with the atmosphere [mm].

Different types of cement result in different degrees of hydration. Creep of concrete depends on the degree of hydration reached at a given age rather than on the age of concrete. Therefore, the effect of type of cement can be taken into account in the new Model Code 2010 by modifying the age at loading \( t_0 \) to \( t_{0,adj} \).

\[
t_{0,adj} = t_{0,T} \left[ \frac{9}{2 + t_{0,T}^{2.5}} + 1 \right]^\alpha \geq 0.5 \text{ days} \quad \text{(5.38)}
\]

where:

- \( t_{0,T} \) is the adjusted age of concrete at loading in days when there are substantial deviations from the mean concrete temperature of 20 °C for the range of 0 °C to +80 °C.
- \( \alpha \) is a coefficient which depends on the type of cement:
  - \( \alpha = -1 \) for strength class 32.5 N;
  - \( \alpha = 0 \) for strength classes 32.5 R, 42.5 N;
  - \( \alpha = 1 \) for strength classes 42.5 R, 52.5 N, 52.5 R.

This model also offers the possibility to be used for stress levels in the range of 0.4 \( f_{cm}(t_0) \leq \sigma_c \leq 0.6 \ f_{cm}(t_0) \), by introducing the non-linear notional creep coefficient \( \varphi_c(t, t_0) \), which replaces \( \varphi(t, t_0) \).

### 5.3.1.4 Shrinkage

The total shrinkage \( \varepsilon_{cs}(t, t_s) \) is subdivided into autogenous \( \varepsilon_{cas}(t) \) and drying shrinkage \( \varepsilon_{cds}(t) \):

\[
\varepsilon_{cs}(t, t_s) = \varepsilon_{cas}(t) + \varepsilon_{cds}(t, t_s) \quad \text{(5.39)}
\]

The autogenous shrinkage can be calculated as follows:

\[
\varepsilon_{cas}(t) = \varepsilon_{cas0}(f_{cm}) \cdot \beta_{as}(t) \quad \text{[14]} \quad \text{(5.40)}
\]

Where \( \varepsilon_{cas0}(f_{cm}) \) is notional autogenous shrinkage coefficient:

\[
\varepsilon_{cas0}(f_{cm}) = -\alpha_{as} \left( \frac{f_{cm} / 10}{6 + f_{cm} / 10} \right)^{2.5} \cdot 10^{-6} \quad \text{[14]} \quad \text{(5.41)}
\]

and \( \beta_{as}(t) \) is the time function:

\[
\beta_{as}(t) = 1 - \exp(-0.2 \cdot \sqrt{t}) \quad \text{[14]} \quad \text{(5.42)}
\]

\( f_{cm} \) is the mean compressive strength at an age of 28 days in MPa;

\( \alpha_{as} \) is a coefficient dependent on the type of cement, according to Table 5.3.

The drying shrinkage can be calculated as follows:

\[
\varepsilon_{cds}(t, t_s) = \varepsilon_{cds0}(f_{cm}) \cdot \beta_{rd}(RH) \cdot \beta_{cds}(t - t_s) \quad \text{[14]} \quad \text{(5.43)}
\]
where:

\[ \varepsilon_{\text{cds}}(t_{cm}) \] is notional drying shrinkage coefficient:

\[
\varepsilon_{\text{cds}}(t_{cm}) = \left[ (220 + 110 \cdot \alpha_{ds1}) \cdot \exp(-\alpha_{ds2} \cdot t_{cm}) \right] \cdot 10^{-6}
\] [14] ........................................... (5.44)

\( \beta_{\text{ds}}(RH) \) is a coefficient that takes into account the effect of the relative ambient humidity:

\[
\beta_{\text{ds}}(RH) = \begin{cases} 
-1.55 \cdot \left[ 1 - \left( \frac{RH}{100} \right)^{0.1} \right] & \text{for } 40 \leq RH < 99\% \cdot \beta_{s1} \text{ [14]} \end{cases}
\] .................................................. (5.45)

with:

\[
\beta_{s1} = \left( \frac{35}{t_{cm}} \right)^{0.1} \leq 1.0 \text{ [14]} \] ................................................................. (5.46)

\( \beta_{ds}(t-t_s) \) is the function describing the time-development:

\[
\beta_{ds}(t-t_s) = \left( \frac{(t-t_s)}{0.035 \cdot h^2 + (t-t_s)} \right)^{0.5}
\] [14] .................................................. (5.47)

where:

\( \alpha_{ds1}, \alpha_{ds2} \) are coefficients dependent on the type of cement, according to Table 5.3;

\( t_{cm} \) is the mean compressive strength at an age of 28 days in MPa;

\( RH \) is the relative humidity of the ambient atmosphere in %;

\( h = 2Ac/u \) = notional size of member [mm], where \( Ac \) is the cross section [mm²] and \( u \) is the perimeter of the member in contact with the atmosphere [mm].

\( t \) is the concrete age in days;

\( t_s \) is the concrete age at the beginning of drying in days;

\( (t-t_s) \) is the duration of drying in days.

**Table 5.3: Coefficients dependent on the type of cement \( \alpha_{ds}, \alpha_{ds1}, \alpha_{ds2} \) [14]**

<table>
<thead>
<tr>
<th>Strength class of cement</th>
<th>( \alpha_{ds} )</th>
<th>( \alpha_{ds1} )</th>
<th>( \alpha_{ds2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>32.5 N</td>
<td>800</td>
<td>3</td>
<td>0.013</td>
</tr>
<tr>
<td>32.5 R, 42.5 N</td>
<td>700</td>
<td>4</td>
<td>0.012</td>
</tr>
<tr>
<td>42.5 R, 52.5 N, 52.5 R</td>
<td>600</td>
<td>6</td>
<td>0.012</td>
</tr>
</tbody>
</table>

### 5.3.2 Model B3 [4]

#### 5.3.2.1 General

Model B3 represents a creep and shrinkage prediction model for analysis and design of concrete structures which was developed by Zdenek P. Bazant and Sandeep Baweja and approved by the ACI Committee 209 in 1995. It is actually an improved alternative of the ACI 209 model, which was developed in the mid-1960's. The model complies with the general guidelines formulated by RILEM TC-107 and is calibrated by a computerized data bank comprising all the relevant test data obtained in various laboratories throughout the world. Advanced knowledge and
improved understanding of the physical processes involved in creep and shrinkage, such as aging, diffusion processes, thermally activated processes, microcracking and their mathematical modeling, as well as development of computerized statistical procedures for data fitting and optimization are incorporated in this model. The coefficient of variation of errors in the predictions of creep and shrinkage strains, calculated by many authors, shows 23% for basic and drying creep and 34% for shrinkage in the case of Model B3, 58% for basic, 45% for drying creep and 55% for shrinkage in the case of the ACI 209 model and 35% for basic, 32% for drying creep and 46% for shrinkage in the case of the CEB-FIP Model Code, 1990. Having this in mind, and the possibility of this model to be extended to special types of concrete such as high-strength and fiber-reinforced concretes if the material parameters are calibrated by short-time tests, makes this model one of the most exact and flexible models [4].

5.3.2.2 Range of Applicability and Parameter Ranges

This model is valid for service stress range $\sigma_c \leq 0.45 f_c$, where $f_c$ is the mean cylinder compressive strength at 28 days.

The prediction of the material parameters is restricted to Portland cement concretes cured for at least one day, with the following parameter ranges:

- $0.35 \leq w/c \leq 0.85$;
- $2.5 \leq a/c \leq 13.5$;
- $160 \text{ kg/m}^3 \leq c \leq 720 \text{ kg/m}^3$;
- $17 \text{ MPa} \leq f_c \leq 70 \text{ MPa}$.

Where:
- $w/c$ is water cement ratio by weight;
- $a/c$ is aggregate cement ratio by weight;
- $c$ is cement content in kg/m$^3$;
- $f_c$ is mean cylinder compressive strength at 28 days in MPa.

For constant stress applied at age of concrete $t'$, the strain can be calculated as follows:

$$\varepsilon(t) = J(t,t')\sigma + \varepsilon_{sh}(t) + \alpha \Delta T(t) \quad [4] \text{ ......................................................... (5.48)}$$

where:
- $J(t,t')$ is the compliance function, which is equal to the strain (elastic plus creep strain) at time $t$, caused by a unit uniaxial constant stress applied at age $t'$;
- $\sigma$ is the uniaxial stress;
- $\varepsilon$ is the strain;
- $\varepsilon_{sh}$ is the shrinkage strain;
- $\Delta T(t)$ is the temperature change from the reference temperature at time $t$;
- $\alpha$ is the thermal expansion coefficient.

The compliance function is further decomposed as:
\[ J(t,t') = q_i + C_0(t,t') + C_0(t,t',t') \] [4] ...........................................

where:

- \( q_i \) is the instantaneous initial strain due to unit stress;
- \( C_0(t,t') \) is the compliance function for basic creep (creep at constant humidity, without spreading of the humidity through the material);
- \( C_0(t,t',t') \) is the additional compliance function due to the effect of simultaneous drying.

The creep coefficient \( \phi(t,t') \), which is the most convenient for including creep into structural analysis, should be calculated from the compliance function:

\[ \phi(t,t') = E(t')J(t,t') - 1 \] [4] ................................................................. (5.50)

where:

- \( E(t') \) – is the static modulus of elasticity at loading age \( t' \)

### 5.3.2.3 Basic Creep

The basic creep compliance is better to be defined by its time rate than by its value:

\[ \dot{C}_0(t,t') = \frac{n(q_2 t^{-m} + q_3)}{(t-t')^{1-m} + q_4 t}, \quad m = 0.5, \quad n = 0.1, \quad [4] \] ................................................... (5.51)

where:

- \( \dot{C}_0(t,t') = \frac{\partial C_0(t,t')}{\partial t} \) [4]...........................................
- \( t \) and \( t' \) are expressed in days;
- \( m \) and \( n \) are the empirical parameters, whose value can be taken the same for all normal concretes as it is shown above;
- \( q_2, q_3 \) and \( q_4 \) are the empirical constitutive parameters.

The total basic creep compliance is obtained by integration of the previous equation:

\[ C_0(t,t') = q_i Q(t,t') + q_i \ln\left[1 + (t-t')^n\right] + q_i \ln\left(\frac{t}{t'}\right) \] [4] ............................................... (5.53)

where \( Q(t,t') \) can be taken from the table or can be calculated with an approximate explicit equation with a smaller error of 1% for \( m=0.5 \) and \( n=0.1 \):

\[ Q(t,t') = Q_i(t') \left[1 + \left(\frac{Q_i(t') Z(t,t')}{Z(t,t')}\right)^{2 \ln(t')}ight]^{\ln(t') \ln(t')} \] [4] ........................................... (5.54)

where:

- \( r(t') = 1.7(t')^{0.12} + 8 \) [4] .............................................................. (5.55)
- \( Z(t,t') = (t')^{-m} \ln\left[1 + (t-t')^n\right] \) [4] ............................................ (5.56)
- \( Q_i(t') = \left[0.086(t')^{0.79} + 1.21(t')^{0.69}\right]^{-1} \) [4] ........................................ (5.57)
5.3.2.4 Shrinkage

The mean shrinkage strain in the cross section is given as follows:

$$\varepsilon_{shw} (t, t_0) = -\varepsilon_{sh} k_h S(t)$$ [4] ................................................................. (5.58)

where the time dependence is expressed with the following equation:

$$S(t) = \tanh \left[ \frac{t - t_0}{\tau_{sh}} \right]$$ [4] ................................................................. (5.59)

The humidity dependence is introduced with the following factor:

$$k_h = \begin{cases} 1 - h^3 & \text{for } h \leq 0.98 \\ -0.2 & \text{for } h = 1 \\ \text{linear interpolation} & \text{for } 0.98 \leq h \leq 1 \end{cases}$$ [4] .................................................. (5.60)

The dependence on the size of the cross-section is given with the following expression:

$$\tau_{sh} = k_s (k_s D)^2$$ [4] ................................................................. (5.61)

where:

- $v/s$ is the ratio between the volume and the surface of the concrete element;
- $D=2v/s$ is the effective cross-section thickness which is equal to the actual thickness in the case of a slab;
- $k_t$ is a factor which is defined with Eq. 5.71:
- $k_s$ is the cross-section shape factor:

$$k_s = \begin{cases} 1.00 & \text{for an infinite slab} \\ 1.15 & \text{for an infinite cylinder} \\ 1.25 & \text{for an infinite square prism} \\ 1.30 & \text{for a sphere} \\ 1.55 & \text{for a cube} \end{cases}$$

For simplified analysis that does not require high accuracy, it can be assumed that $k_s=1$.

The time dependence of the ultimate shrinkage strain can be expressed as it follows:

$$\varepsilon_{shw} = \varepsilon_w \left( \frac{E(607)}{E(t_0 + \tau_{sh})} \right) E(t) = E(28) \left( \frac{t}{4 + 0.85t} \right)^{1/2}$$ [4] ................................................................. (5.62)

where:

- $\varepsilon_w$ is a constant which is given with Eq. 5.70. This means that $\varepsilon_w = \varepsilon_{shw}$ for $t_0=7$ days and $\tau_{sh}=600$ days.

5.3.2.5 Drying Creep

$$C_0 (t, t', t_0) = q_s \left[ \exp \left[ -8H(t) \right] - \exp \left[ -8H(t_0') \right] \right]^{1/2}, t_0' = \max (t', t_0)$$ if $t \geq t_0'$ [4] ............... (5.63)

otherwise $C_0 (t, t', t_0) = 0$,

$t_0'$ is the time at which drying and loading first act simultaneously; and

$$H(t) = 1 - (1 - h) S(t)$$ [4] ................................................................. (5.64)
5.3.2.6 Model Parameters

The model parameters are estimated on the basis of the concrete strength and composition, or more precisely, by the water-cement ratio, the aggregate-cement ratio, the mean cylinder compressive strength and the modulus of elasticity.

- Basic creep

\[ q_1 = 0.6 \times 10^6 / E_{28} \] \[ E_{28} = 4734 \sqrt{f_c} \] \[ q_2 = 185.4c^{-0.5} f_c^{-0.9} \] \[ q_3 = 0.29(w / c)^0.6 q_2 \] \[ q_4 = 20.3(a / c)^0.7 q_2 \]

- Shrinkage

\[ \epsilon_{aw} = -\alpha_1 \alpha_2 \left[ 1.9 \times 10^{-2} w^{-1} f_c^{-0.28} + 270 \right] \times 10^{-6} \]

\[ K_7 = 8.5t_0^{-0.08} f_c^{-1/4} \text{ days/cm}^2 \]

where:

\[ \alpha_1 = \begin{cases} 1.0 & \text{for type I cement;} \\ 0.85 & \text{for type II cement;} \\ 1.1 & \text{for type III cement.} \end{cases} \]

\[ \alpha_2 = \begin{cases} 0.75 & \text{for steam curing;} \\ 1.2 & \text{for sealed or normal curing in air with initial protection against drying;} \\ 1.0 & \text{for curing in water or at 100\% relative humidity.} \end{cases} \]

- Drying creep

\[ q_5 = 7.57 \times 10^4 f_c^{-1} \left[ \epsilon_{aw} \right]^{-0.6} \]

The model offers the possibility of calculating the autogenous shrinkage, which is caused by the chemical reactions of hydration and can be neglected for normal strength concretes, but must be taken into account when one deals with high strength concretes. In that case, the total shrinkage is decomposed to autogenous and drying shrinkage:

\[ \epsilon_{sh}^{\text{total}}(t,t_0) = \epsilon_a(t) + \epsilon_{sh}(t,t_0) \]

where:

\[ \epsilon_a \] is the autogenous shrinkage strain;
\[ \epsilon_{sh} \] is the drying shrinkage strain.

The value of the autogenous shrinkage strain can be calculated with the following equation:

\[ \epsilon_a(t) = \epsilon_{aw}(0.99 - h_\infty)S_a(t) \]

\[ S_a(t) = \tanh \left[ \frac{t - t_0}{\tau_a} \right] \]

where:
\( t_s \) is the time of the final set of the cement;  
\( \varepsilon_{sh} \) is the drying shrinkage;  
\( t_a \) is the half time of the autogenous shrinkage which depends on the level of hardening of the high strength concrete;  
\( h_{\infty} \) is the final self-desiccation humidity (which can be assumed as 80%).

### 5.4 Stress-Strain Relation for Reinforcement

The stress-strain relation for reinforcement is also necessary to be defined in structural analysis of reinforced concrete sections.

In EC2, the behavior of the reinforcement is defined by the following properties [13]:
- yield strength \( (f_{yk} \text{ or } f_{0.2k}) \);
- maximum actual yield strength \((f_{y,max})\);
- tensile strength \( (f_t)\);
- ductility \((\varepsilon_{uk} \text{ and } f_t/f_{yk})\);
- bendability;
- bond characteristics;
- section sizes and tolerances;
- fatigue strength;
- weldability;
- shear and weld strength for welded fabric and lattice girders.

The application rules for design and detailing in Eurocode 2 are valid for the specified yield strength range of \( f_{yk}=400-600 \text{MPa} \).

The yield strength \( f_{yk} \) (or the 0.2\% proof stress, \( f_{0.2k} \)) and the tensile strength \( f_t \) are defined respectively as the characteristic value of the yield load and the characteristic maximum load in direct axial tension, each divided by the nominal cross sectional area.

Figure 5.7 shows the typical stress-strain diagrams for hot rolled and cold worked steel [13].

![Figure 5.7: Stress-strain diagrams of typical reinforcing steel: a) hot rolled steel; b) cold worked steel [13]](image-url)
The design should be based on the nominal cross-section area of the reinforcement and the design values derived from characteristic values. For usual design of reinforced concrete sections, either of the following assumptions may be made, as presented also in Figure 5.8 [13]:

- An inclined top branch with a strain limit of $\varepsilon_{ud}$ and a maximum stress of $k f_{y}/\gamma_s$ at strain $\varepsilon_{uk}$, where $k=(f/f_{y})_{k}$ [13]
- A horizontal top branch that does not necessitate checking of the strain limit [13]

The recommended value for $\varepsilon_{ud}$ is $0.9\varepsilon_{uk}$ [13]. The value of the characteristic strain under the effect of maximum stress $\varepsilon_{ud}($%) for reinforcing steel class A, B and C is $\geq2.5; \geq5; \geq7.5$, respectively [13].

![Idealised and design stress-strain relations for reinforcing steel](image)

Figure 5.8: Idealised and design stress-strain relations for reinforcing steel (for tension and compression) [13]
CHAPTER 6: METHODS FOR STRUCTURAL ANALYSIS OF CREEP

6.1 General

Time-dependent analysis of a reinforced concrete structure involves calculation of strains, stresses, curvatures and deflections at critical sections and at critical times during the life of the structure. Very often, the structural designer is interested in the final deformation and final internal actions at time infinity, after the reaching of almost final value of creep and shrinkage and redistribution of the stresses between concrete and reinforcement. If sustained load is applied on the structure, instantaneous strain followed by creep and shrinkage strains occur. Things become complicated if the applied load varies with time, or even if it is constant, the stress in concrete is rarely constant. In addition, concrete is an aging material which changes its properties with time. Therefore, the stress history and the effect of aging must be included in the time-dependent analysis of concrete structures.

Many, more or less simplified methods for structural analysis of creep are available in literature. The oldest and simplest method for including creep in structural analysis is the Effective Modulus Method (EMM) proposed by McMillan in 1916 [17]. In this method, the creep is treated as a delayed elastic strain and it is taken into account simply by reducing the elastic modulus of concrete with time. In this way, the time-dependent analysis using the EMM is actually an elastic analysis in which the modulus of elasticity is replaced by effective modulus of elasticity (Eq. 6.2).

The total strain in concrete at time $t$ with this method can be obtained as previously stated in Eq.5.21, by summing the stress-produced strains and the shrinkage strain:

$$\varepsilon(t) = \int_{t_0}^{t} \frac{1 + \varphi(t, \tau)}{E_c(\tau)} d\sigma_c(\tau) + \varepsilon_{sh}(t) = \frac{1 + \varphi(t, \tau_0)}{E_c(\tau_0)} \sigma_c(t) + \varepsilon_{sh}(t) = \frac{\sigma_c(t)}{E_o(t, \tau_0)} + \varepsilon_{sh}(t) \quad [16] \quad \text{(6.1)}$$

Eq.5.21

Where the effective modulus of elasticity is defined as follows:

$$E_o(t, \tau_0) = \frac{E_c(\tau_0)}{1 + \varphi(t, \tau_0)} \quad [16] \quad \text{...............(6.2)}$$

EMM is valid only if the concrete stress is constant in time.

With EMM, the creep strain at time $t$ depends only on the current stress in the concrete $\sigma_c(t)$ and is independent of the previous stress history. The aging of concrete is also ignored. If the stress is completely removed, the creep strains disappear, or complete creep recovery occurs, which is incorrect [16].

Other more sophisticated methods which were later developed are the rate of creep method (Glanville-Dischinger method, 1933-1939), the improved Dischinger method (Ruesch, Jungwirth and Hilsdorf, 1973), the rate of flow method (England and Illston, 1965) or the Maslov-Arutyunyan method (Maslov, 1941 and Arutyunyan, 1952). These methods use a simplified creep compliance function as the basis for simplification of the structural analysis of creep [17].
Chapter 6 METHODS FOR STRUCTURAL ANALYSIS OF CREEP

6.2 Age-Adjusted Effective Modulus Method (AAEMM)

The development of the AAEMM started with a simple adjustment of the EMM to account for the aging of concrete, proposed by Trost in 1967. The method was later developed by Dilger and Neville and mathematically proven and formulated by Bazant (1972). It is sometimes called the Trost-Bazant Method.

The AAEMM can easily be understood if two different concrete stress histories and the corresponding creep-time curves are considered, as presented in Figure 6.1. In the first stress history (a), the stress is applied at time $\tau_0$ and is kept constant up to time $\tau_k$. In the second stress history (b), the stress is gradually applied, beginning at time $\tau_0$ and reaching its final value at time $\tau_k$. The creep strain which occurs due to the application of the load at any time $t$ is much smaller for the gradually increased load history than for the constant stress history. This is due to aging and the fact that, in the first load history, the specimen is loaded at an earlier age, and therefore, greater creep strain is obtained. Having this in mind, a reduced creep coefficient can be used to calculate the creep strain. The reduced creep coefficient is denoted as $\chi(t, \tau_0) \varphi(t, \tau_0)$, where $\chi(t, \tau_0)$ is called the aging coefficient [16].

The creep strain at any time $t$, due to stress $\sigma_c(t)$ that has gradually been applied over the time interval $t - \tau_0$, can be calculated as follows:

$$\varepsilon_{cr}(t) = \frac{\sigma_c(t)}{E_c(\tau_0)} \chi(t, \tau_0) \varphi(t, \tau_0) \quad [16]$$

With this method, two analyses need to be carried out, one at first loading and one at time $t$, after the period of sustained stress.

The aging coefficient depends on the age of the concrete at first loading, the duration of the load, the size and the shape of the specimen, etc. It is usually in the range of 0.4-1.0. Its magnitude varies with time, but for practical calculations where only the final deformation is of importance, a constant value can be assumed. For creep problems (constant load) referring to specimens loaded at ages earlier than 20 days, it can be assumed that $\chi(t, \tau_0) = 0.65$, while for concrete loaded at ages later than 28 days, it can be assumed that $\chi(t, \tau_0) = 0.75$ [16].

![Figure 6.1: Creep due to constant and variable stress histories [16]](image-url)
The total strain at time \( t \) may be expressed as the sum of the strains produced by
\( \sigma_z(t, \tau_o) \) (instantaneous and creep), the strains produced by the gradually applied stress increment
\( \Delta \sigma_z(t) \) (instantaneous and creep), and the shrinkage strain:

\[
\varepsilon(t) = \frac{\sigma_z(t, \tau_o)}{E_z(\tau_o)} \left[ 1 + \phi(t, \tau_o) \right] + \frac{\Delta \sigma_z(t)}{E_z(\tau_o)} \left[ 1 + \chi(t, \tau_o) \phi(t, \tau_o) \right] + \varepsilon_{sh}(t) = \frac{\sigma_z(t, \tau_o)}{E_z(t, \tau_o)} + \frac{\Delta \sigma_z(t)}{E_z(t, \tau_o)} + \varepsilon_{sh}(t) \quad \text{(6.4)}
\]

where \( E_z(t, \tau_o) \) is the effective modulus of elasticity and \( \overline{E_z}(t, \tau_o) \) is the age-adjusted effective
modulus of elasticity and can be expressed by:

\[
\overline{E_z}(t, \tau_o) = \frac{E_z(\tau_o)}{1 + \chi(t, \tau_o) \phi(t, \tau_o)} \quad \text{(6.5)}
\]

### 6.3 Age-Adjusted Effective Modulus Method (AAEMM) for SFRC

The instantaneous midspan deflection \( a_i \) of a simply supported steel fibre reinforced concrete
beam with span \( l \), loaded with uniform load \( q \) and concentrated forces \( F \), at a distance \( x \) from the
supports, can be calculated as follows:

\[
a_i = \frac{5}{384} \frac{q l^4}{E_s I_s} + \frac{F x}{24E_s I_s} \left( 3l^2 - 4x^2 \right) \quad \text{(6.6)}
\]

where:

- \( E_s \) is the modulus of elasticity of steel fibre reinforced concrete,
- \( I_s \) is the moment of inertia which, according to Branson, can be calculated as:

\[
I_s = \left( \frac{M_{cr}}{M_a} \right)^{\frac{3}{2}} I_g + \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^{\frac{3}{2}} \right] I_g \quad \text{(6.7)}
\]

where:

- \( M_{cr} \) is the cracking moment,
- \( M_a \) is the maximum applied moment,
- \( I_g \) is the gross moment of inertia, and
- \( I_g \) is the moment of inertia of a transformed cracked SFRC beam section. According to
Tan, Paramasivam and Tan [28], it can be calculated as:

\[
I_g = \frac{b x^3}{3} + n A_t \frac{(d - x)^2}{2} + (n - 1) A_t \frac{(x - d')^2}{3} + n A_t \frac{(h - x)^2}{3} + (n - 1) A_t \frac{x^2}{3} \quad \text{(6.8)}
\]

where:

- \( b \) is the width of the beam,
- \( h \) is the height of the beam,
- \( d \) is the distance from the compression face to the centroid of the tensile reinforcement,
- \( d' \) is the distance from the compression face to the centroid of the compressive
reinforcement,
- \( A_t \) is the area of the tensile reinforcement,
\( A_t \) is the area of the compressive reinforcement,
\( n = E_s / E_d, n_r = E_r / E_d \) are the ratios between the modulus of elasticity of the reinforcing steel and steel fibers on one hand and the modulus of elasticity of the steel fibre reinforced concrete on the other hand,
\( x \) is the position of the neutral axis, and
\( A_t \) and \( A_r \) are areas that are taking into account the steel fibers and can be calculated as follows:
\[
A_t = \eta \eta_b V_r b (h - x) \quad \text{[28]} \] \hspace{1cm} (6.9)
\[
A_r = \eta \eta_b V_r b x \quad \text{[28]} \] \hspace{1cm} (6.10)

Actually, the last two terms in Eq.6.8 refer to the second moment of area of steel fibers in the tensile and compression zones, taken about the neutral axis of the transformed cracked section. The steel fibers are introduced in Eq.6.9 and Eq.6.10 by their volume fraction \( V_r \), by the length efficiency factor \( \eta \) and by the orientation factors before and after cracking \( \eta_0 \) and \( \eta_0' \).

The deflection due to creep can be determined by Eq.6.6 if \( E_d, v \) is replaced by \( E_{cr}, I_{cr} \) calculated by Eq.6.5 and the effective moment of inertia at time \( t \) with the corresponding moment of inertia of the transformed cracked section at time \( t \) are calculated from Eq.6.11 and Eq.6.12.

\[
I_{t,cr} = \left( \frac{M_{cr}}{M_t} \right)^3 I_0 + \left[ 1 - \left( \frac{M_{cr}}{M_t} \right)^3 \right] I_{cr,cr} \leq I_0 \quad \text{[6.11]} \]

\[
I_{cr,cr} = \frac{b x^3}{3} + n_t A_x (d - x)^2 + (n_r - 1) A_x \left( x - d' \right)^2 + n_n A_y \left( h - x_i \right)^2 + (n_n - 1) A_y \frac{x_i^2}{3} \quad \text{[6.12]} \]

The values of \( n \) and \( n_r \) are time-dependent due to the change of the modulus of elasticity with time as well as the position of the neutral axis \( x \). If the position of the neutral axis is unknown, it can be calculated based on the equilibrium of forces and strain compatibility, taking into account the beneficial effect of the steel fibers.

The deflection due to shrinkage of a concrete member with span \( l \) can be calculated from the following equation:

\[
a_{sh} = \frac{l}{6} \phi_{sh}^2 \quad \text{[6.13]} \]

where:

\( \phi_{sh} \) is the curvature due to shrinkage of concrete in an asymmetrically reinforced concrete element and can be calculated as:

\[
\phi_{sh} = \frac{T_s e}{E' I_{cr,cr}} \quad \text{[6.14]} \]
Where \( e \) is the distance between the centroid of the uncracked transformed area and the steel area and \( T_s \) is a fictitious tensile force calculated by Eq. 6.15.

\[
T_s = (A_s + A_{s}')(\varepsilon_{sn}(t,t')E_s)
\]

where \( \varepsilon_{sn}(t,t') \) is the free shrinkage strain of the steel fiber reinforced concrete.
CHAPTER 7: EXPERIMENTAL PROGRAM

7.1 Objective of the Experiment

Because of the scarcity of investigations in the area of long-term behaviour of steel fibre reinforced concrete elements, the objective of this experiment has been to contribute to the creation of a data base, which will contribute to definition of a computational model that will present the influence of sustained and repeated variable load, as well as the effects of creep and shrinkage on time-dependent behaviour of SFRC elements. The experiment was carried out at the “Ss. Cyril and Methodius” University, Faculty of Civil Engineering in Skopje, Republic of Macedonia, in the period October 2011 to February 2013.

7.2 Description of the Experimental Program

The experiment involved testing of 24 full scale beams constructed from reinforced concrete and steel fibre reinforced concrete with additional reinforcement. The beams had a cross section proportioned 15/28cm and a total length of l=300cm, Figure 7.1. Together with each series of beams, control specimens were cast in order to test the compressive strength, flexural tensile strength, splitting tensile strength, elastic modulus and deformations due to creep and shrinkage. In addition to the tests on mechanical and time-dependent properties of concrete, the used reinforcement was also tested.

Figure 7.1: Geometry, reinforcement and loading scheme of full scale beams

All 24 beams were manufactured with concrete class C30/37. According to the used type of material, they were divided into three series:
- Series A, reinforced concrete (C30/37);
- Series B, SFRC with 30 kg/m³ steel fibres and additional reinforcement (C30/37 FL1.5/1.5);
- Series C, SFRC with 60 kg/m³ steel fibres and additional reinforcement (C30/37 FL2.5/2.0).

The beams constructed of reinforced concrete were used for comparison with the beams constructed of steel fibre reinforced concrete.

In each series, the plain reinforcement was kept the same. The longitudinal reinforcement was ribbed and of RA 400/500-2 quality, while the shear reinforcement was smooth, with GA 240/360 quality. Reinforcement 2Ø10, 2Ø8 and Ø6/10/20cm was used as tension, compression and shear reinforcement, respectively.
In order to find out the influence of different fibre dosages on the behavior of the elements with time, the investigated parameter in this research was the fibre dosage. The used steel fibres were hooked-end HE1/50, manufactured by Arcelor Mittal, with a diameter of 1mm, length of 50mm and tensile strength of 1100 N/mm². The fibres were produced of cold-drawn wire and are presented in Figure 7.2.

![Technical Data Sheet of Used Fibres](image)

**Figure 7.2: Technical data sheet of used fibres**

The mixture proportioning was done so that it was the same for the three types of concrete. It is presented in Table 7.1.

**Table 7.1: Mixture proportions for C30/37, C30/37 FL1.5/1.5 and C30/37 FL2.5/2.0**

<table>
<thead>
<tr>
<th>Mixture proportions</th>
<th>(kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement CEM II/A-M 42.5N</td>
<td>410</td>
</tr>
<tr>
<td>Water</td>
<td>215</td>
</tr>
<tr>
<td>Water/Cement ratio, w/c</td>
<td>0.524</td>
</tr>
<tr>
<td>Aggregate:</td>
<td></td>
</tr>
<tr>
<td>0-4 mm (river sand), 50%</td>
<td>875</td>
</tr>
<tr>
<td>4-8 mm (limestone), 20%</td>
<td>350</td>
</tr>
<tr>
<td>8-16 mm (limestone), 30%</td>
<td>525</td>
</tr>
<tr>
<td>Fibres:</td>
<td></td>
</tr>
<tr>
<td>C30/37</td>
<td>0</td>
</tr>
<tr>
<td>C30/37 FL 1.5/1.5</td>
<td>30</td>
</tr>
<tr>
<td>C30/37 FL 2.5/2.0</td>
<td>60</td>
</tr>
</tbody>
</table>

Regarding the loading history, the beams were divided into four groups:

1. The beams from all three series from group "1" (A₁, B₁, C₁) were tested under short term ultimate load at the age of concrete of 40 days (Figure 7.3). With these testing, relevant
dependences had to be found for this age of concrete and the behavior of the reinforced concrete and two types of steel fiber reinforced concretes had to be compared.

2. The beams from all three series from group “2” (A₂, B₂, C₂) were tested also under short term ultimate load, but at the age of concrete of 400 days (Figure 7.3). This testing was performed in order to find out the influence of the age of concrete on the behavior of the beams.

3. The beams from group “3” (A₃, B₃, C₃) were pre-cracked with permanent and variable load “g + q”, and afterwards, a long term permanent load with intensity “g” was applied at the age of concrete of 40 days and was held up to 400 days, when a short term ultimate load testing was performed (Figure 7.4). In the meantime, the strains, deformations and crack widths were measured.

4. On the beams from group “4” (A₄, B₄, C₄), a long term permanent load with intensity “g” was applied at the age of concrete of 40 days and was held for a year as a long term load. On the fortieth day, variable repeated load “± q” was also applied in an interval of 8 hours +q and 16 hours –q, for a year (Figure 7.4). This means that the beams were loaded additionally with load “q” for 8 hours every day, whereat the strains, deformations and crack widths were measured. After 8 hours, the beams were unloaded from load “q” and all measurements were performed again. This procedure was repeated every day for a year in order to simulate a realistic load history.

The experimental program is presented in detail in Table 7.2.
### Table 7.2: Experimental program

<table>
<thead>
<tr>
<th>Series</th>
<th>Group</th>
<th>Number of elements</th>
<th>Type of concrete</th>
<th>Steel fibres (kg/m³)</th>
<th>Tensile Reinforcement μ (%)</th>
<th>Type of long term load</th>
<th>Time of ultimate load testing</th>
<th>Time of observing of the elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1 2</td>
<td>C30/37</td>
<td>0</td>
<td>0.37</td>
<td>/</td>
<td>/</td>
<td>t=40</td>
<td>t=360</td>
</tr>
<tr>
<td></td>
<td>2 2</td>
<td>C30/37</td>
<td>0</td>
<td>0.37</td>
<td>&quot;g&quot; *</td>
<td>/</td>
<td>t=400</td>
<td>t=360</td>
</tr>
<tr>
<td></td>
<td>3 2</td>
<td>C30/37</td>
<td>0</td>
<td>0.37</td>
<td>&quot;g ± q&quot; (Δtg+q=8h)</td>
<td>/</td>
<td>t=400</td>
<td>t=360</td>
</tr>
<tr>
<td></td>
<td>4 2</td>
<td>C30/37</td>
<td>0</td>
<td>0.37</td>
<td>/</td>
<td>/</td>
<td>t=400</td>
<td>t=360</td>
</tr>
<tr>
<td>B</td>
<td>1 2</td>
<td>C30/37 FL 1.5/1.5</td>
<td>30</td>
<td>0.37</td>
<td>/</td>
<td>/</td>
<td>t=40</td>
<td>t=360</td>
</tr>
<tr>
<td></td>
<td>2 2</td>
<td>C30/37 FL 1.5/1.5</td>
<td>30</td>
<td>0.37</td>
<td>&quot;g&quot; *</td>
<td>/</td>
<td>t=400</td>
<td>t=360</td>
</tr>
<tr>
<td></td>
<td>3 2</td>
<td>C30/37 FL 1.5/1.5</td>
<td>30</td>
<td>0.37</td>
<td>&quot;g ± q&quot; (Δtg+q=8h)</td>
<td>/</td>
<td>t=400</td>
<td>t=360</td>
</tr>
<tr>
<td></td>
<td>4 2</td>
<td>C30/37 FL 1.5/1.5</td>
<td>30</td>
<td>0.37</td>
<td>/</td>
<td>/</td>
<td>t=400</td>
<td>t=360</td>
</tr>
<tr>
<td>C</td>
<td>1 2</td>
<td>C30/37 FL 2.5/2.0</td>
<td>60</td>
<td>0.37</td>
<td>/</td>
<td>/</td>
<td>t=40</td>
<td>t=360</td>
</tr>
<tr>
<td></td>
<td>2 2</td>
<td>C30/37 FL 2.5/2.0</td>
<td>60</td>
<td>0.37</td>
<td>&quot;g&quot; *</td>
<td>/</td>
<td>t=400</td>
<td>t=360</td>
</tr>
<tr>
<td></td>
<td>3 2</td>
<td>C30/37 FL 2.5/2.0</td>
<td>60</td>
<td>0.37</td>
<td>&quot;g ± q&quot; (Δtg+q=8h)</td>
<td>/</td>
<td>t=400</td>
<td>t=360</td>
</tr>
<tr>
<td></td>
<td>4 2</td>
<td>C30/37 FL 2.5/2.0</td>
<td>60</td>
<td>0.37</td>
<td>/</td>
<td>/</td>
<td>t=400</td>
<td>t=360</td>
</tr>
</tbody>
</table>

C30/37 - reinforced concrete; C30/37 FL 1.5/1.5 - Steel fibre reinforced concrete with 30 kg/m³ (0.38%) steel fibres and additional reinforcement 2010; C30/37 FL 2.5/2.0 - Steel fibre reinforced concrete with 60 kg/m³ (0.76%) steel fibres and additional reinforcement 2010; Δt=8h (8h under g+q, 16h under g).

"pre-cracked with load "g + q"

The casting of the beams (Figure 7.5) was done by a series of 8 beams in wooden moulds in the “Karposh” a.d factory - Skopje, in October and November 2011. Each time, 42 control specimens (Figure 7.5) were cast for testing of the mechanical and time-dependent characteristics of concrete at the age of 40 and 400 days. The beams and control specimens were cured for 8 days and then they were transported to the Laboratory at the Faculty of Civil Engineering – Skopje (Figure 7.6), where they were kept under almost constant temperature ranging from 15-24°C with an average of 19.5°C and constant relative ambient humidity ranging from 58-62% with an average of 60.2%, which was regulated with special humidifiers and dehumidifiers (Figure 7.6). The temperature and humidity through the period of 400 days are
Chapter 7 EXPERIMENTAL PROGRAM

presented in Figure 7.7.

Figure 7.5: Casting of beams and control specimens

Figure 7.6: Laboratory of the Faculty of Civil Engineering – Skopje

Figure 7.7: Ambient conditions in the Laboratory of the Faculty of Civil Engineering – Skopje

PhD Thesis: Time-Dependent Behaviour of SFRC Elements under Sustained and Repeated Variable Loads
7.3 Application of Long Term Load

The long term load, which consists of permanent sustained load “g” and variable repeated load “q”, was applied by 12 gravitation levers (Figure 7.8), which enabled an increase of the load for 13 times. The permanent load acted all the time, while the variable load was applied and removed each day by secondary hand gravitation levers.

The bending moments were as follows: from self-weight of the beam, \( M_{sw}=1 \text{kNm} \), from permanent load “g”, \( M_g=5.0 \text{kNm} \), from variable load “q”, \( M_q=3.1 \text{kNm} \), from self-weight, permanent and variable load (service) \( M_{sw+g+q}=9.1 \text{kNm} \). The bending crack moment was \( M_{cr}=6.1 \text{kNm} \), while the ultimate bending moment was \( M_d=15.6 \text{kNm} \). The intensity of the load was chosen so that \( M_{cr} \) was bigger than \( M_{sw+g} \) and smaller than \( M_{sw+g+q} \). The permanent load was 0.39 times the flexural strength, while the service load was 0.58 times the flexural strength of the beam without fibres.

The transferred forces from the gravitation levers to the beams were measured and controlled by a dynamometer for the above mentioned load levels.

![Figure 7.8: Gravitation lever](image)

7.4 Measurement Technique

On the 12 full scale beams within all three series from group 1 and 2, short term ultimate load test was performed in 29 load steps. A data acquisition system from Hottinger Baldvin-HBM, Germany was used for recording the force, the middle deflection \( U_3 \), the strains in the compression and tensile reinforcement (R1-R4) and the bottom and top of the concrete (C1-C4), with \( f=1 \text{Hz} \). The load cell and the LVT as well as the strain gages (Figure 7.10) were a product of Kyowa, Japan. In each step, the strains in the concrete (D1-D15), in the middle section of the beam through the thickness as well as on the top of the beam, were measured by a mechanical deflection meter, type Hugenerberger, Switzerland, with a base of 250mm. The mechanical measurement of the deflections was done at 5 points through the length of the beam and 2 points over the supports by using deflection meters produced by Stopani, Italy. The crack widths were also measured in each load step, in the region with constant moment, by use of a crack width meter.
As to the other 12 full scale beams within all three series from groups 3 and 4, after 360 days of observing their behaviour under the effect of long term sustained and repeated variable load, an ultimate load test was performed in the same manner as described above. The same measurement technique was used during the observation of the long term behaviour. The positions of the measurement points of the full scale beams are presented in Figure 7.9.

Figure 7.9: Position of measurement points of full scale beams
7.5 Testing of Control Specimens

The mechanical properties of the three series were tested at the age of 40 days when the full scale beams were loaded, each according to the previously specified loading history. The mechanical characteristics were also tested at the age of 400 days when ultimate load testing was performed on all beams that were subjected to long term sustained and repeated variable load.

The mechanical properties (Figure 7.11) at the age of 40 days were tested on 3 specimens for compressive strength, splitting tensile strength and Modulus of Elasticity and 6 specimens for flexural tensile strength, according to RILEM TC 162-TDF [26]. The mechanical properties the age of 400 days were tested also on 3 specimens for compressive strength, splitting tensile strength and Modulus of Elasticity and 3 specimens for flexural tensile strength. The compressive strength, splitting tensile strength and Modulus of Elasticity were tested by use of hydraulic jack HPM3000, produced by ZRMK - Ljubljana, Slovenia.

The most specific testing was the testing of the flexural tensile strength, which was performed according to RILEM TC 162-TDF [26]. The deflection controlled testing was done on notched prisms with cross section dimensions of 15/15cm, length of 70cm and span of 50cm. Beams were notched by wet sawing with width of notch of 5mm and depth of 25mm. The application of the deflection was at a constant rate of 0.2mm/min by use of Wykeham Farrance, England, which is a 50kN machine with a big stiffness. During the tests, the load and the midspan deflection were
recorded continuously by the data acquisition system produced by Hottinger Baldvin-HBM, Germany.

Figure 7.11: Testing of mechanical properties

Compression creep was applied in creep frames (Figure 7.12) whereat the stress level of the 12x12x36cm prism specimens was the same as the stress in the full scale beams due to the sustained load, which was 7.5MPa. The drying shrinkage was measured immediately after the opening of the moulds of the control specimens (Figure 7.13).

Figure 7.12: Testing of concrete creep in creep frames
Figure 7.13: Measurement of drying shrinkage

Testing of the reinforcement was done on 10 control specimens for each diameter, Ø8 and Ø10, tested by use of the universal hydraulic testing machine produced by A.J.Amsler, Schaffhausen, Switzerland with a capacity of 300kN, in the range of up to 100kN (Figure 7.14).

Figure 7.14: Testing of reinforcement

7.6 Results from Testing of Mechanical and Time-Dependent Deformation Properties

The mixture proportioning was done according to all recommendations [8], [22] and [24] in the up to date literature, so the slump of the concrete without fibres was 120mm. Since fibres decrease workability, the slump was decreased to 75mm and 50mm with addition of 30 and 60kg/m³ (Table 7.3).

The average results for the mechanical properties with the corresponding standard deviations are presented in Table 7.4. The increase in the mechanical characteristics with time, which is...
typical for concrete, is presented in the same table.

Table 7.3: Slump of C30/37, C30/37 FL 1.5/1.5 and C30/37 FL 2.5/2.0

<table>
<thead>
<tr>
<th>Type of concrete</th>
<th>Slump (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reinforced Concrete (C30/37)</td>
<td>120</td>
</tr>
<tr>
<td>Steel fibre reinforced concrete with 30 kg/m³ (C30/37 FL 1.5/1.5)</td>
<td>75</td>
</tr>
<tr>
<td>Steel fibre reinforced concrete with 60 kg/m³ (C30/37 FL 2.5/2.0)</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 7.4: Mechanical properties at age of 40 and 400 days of C30/37, C30/37 FL 1.5/1.5 and C30/37 FL 2.5/2.0

<table>
<thead>
<tr>
<th>Mechanical properties</th>
<th>Age at testing t(days)</th>
<th>C30/37 σ (st.dev)</th>
<th>C30/37 FL 1.5/1.5 σ (st.dev)</th>
<th>C30/37 FL 2.5/2.0 σ (st.dev)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressive strength (MPa) (cubes 15/15/15cm)</td>
<td>40</td>
<td>42.89 0.18</td>
<td>41.63 4.79</td>
<td>44.59 1.83</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>45.70 5.742</td>
<td>47.41 1.07</td>
<td>46.15 1.69</td>
</tr>
<tr>
<td>Increase (%)</td>
<td>6.55</td>
<td>13.88</td>
<td>3.50</td>
<td></td>
</tr>
<tr>
<td>Splitting tensile strength (MPa) (cubes 15/15/15cm)</td>
<td>40</td>
<td>3.51 0.10</td>
<td>3.22 0.14</td>
<td>4.00 0.31</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>4.17 0.02</td>
<td>4.58 0.13</td>
<td>4.24 0.13</td>
</tr>
<tr>
<td>Increase (%)</td>
<td>18.80</td>
<td>42.23</td>
<td>6.00</td>
<td></td>
</tr>
<tr>
<td>Flexural tensile strength (MPa) (beams 15/15/70cm)</td>
<td>40</td>
<td>5.18 0.56</td>
<td>4.95 0.34</td>
<td>5.30 0.66</td>
</tr>
<tr>
<td>- σ₁ (stress at δ₁=0.05mm)</td>
<td></td>
<td>1.80 0.44</td>
<td>2.83 0.67</td>
<td></td>
</tr>
<tr>
<td>- σ₂ (stress at δ₂,1=0.46mm)</td>
<td></td>
<td>1.53 0.40</td>
<td>2.33 0.73</td>
<td></td>
</tr>
<tr>
<td>- σ₃ (stress at δ₃,4=3.00mm)</td>
<td></td>
<td>1.38 0.28</td>
<td>2.90 0.44</td>
<td></td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>5.00 0.66</td>
<td>4.40 1.32</td>
<td>5.95 0.13</td>
</tr>
<tr>
<td>- σ₁ (stress at δ₁=0.05mm)</td>
<td></td>
<td>1.38 0.28</td>
<td>2.90 0.44</td>
<td></td>
</tr>
<tr>
<td>- σ₂ (stress at δ₂,1=0.46mm)</td>
<td></td>
<td>1.15 0.47</td>
<td>2.38 0.32</td>
<td></td>
</tr>
<tr>
<td>- σ₃ (stress at δ₃,4=3.00mm)</td>
<td></td>
<td>1.15 0.47</td>
<td>2.38 0.32</td>
<td></td>
</tr>
<tr>
<td>Increase σ₁ (%)</td>
<td>-3.5</td>
<td>-11.11</td>
<td>12.26</td>
<td></td>
</tr>
<tr>
<td>Modulus of Elasticity (MPa) (cylinders 15/30cm)</td>
<td>40</td>
<td>26956 127.2</td>
<td>26771 93.2</td>
<td>26120 423.2</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>27041 811.4</td>
<td>30809 618.2</td>
<td>28224 674.2</td>
</tr>
<tr>
<td>Increase (%)</td>
<td>0.32</td>
<td>15.08</td>
<td>8.06</td>
<td></td>
</tr>
</tbody>
</table>

The testing of the flexural tensile strength at the age of 40 days resulted in force – deflection relationship for the three types of concrete shown in Figure 7.15. The same relationships for the age of 400 days are presented in Figure 7.16. The results for the concrete type without fibres, i.e., C30/37, are also presented in these two figures. However, it must be noted that, as it usually happens in the case of plain concrete, sudden brittle failure occurred, manifested by a sudden drop in force in less than a second. The resulting stress – strain relationship in tension at the age of 40 days for steel fibre reinforced concretes C30/37 FL 1.5/1.5 and C30/37 FL 2.5/2.0 is presented in Figure 7.17. For the age of 400 days, it is presented in Figure 7.18.
Figure 7.15: Bending test on notched beams at the age of 40 days for the three concrete types.
Figure 7.16: Bending test on notched beams at the age of 400 days for the three concrete types
Figure 7.17: Stress – strain relationship in tension at the age of 40 days.

Figure 7.18: Stress – strain relationship in tension at the age of 400 days.

The stress – strain relationship in compression that resulted from the testing of the Modulus of Elasticity at the age of 40 and 400 days is presented in Figure 7.19 and Figure 7.20.

Figure 7.19: Stress – strain relationship in compression at the age of 40 days for the three concrete types.
Figure 7.20: Stress – strain relationship in compression at the age of 400 days for the three concrete types

The stress – strain relationships in compression for each type of concrete are presented as a mean value of the testing of three control specimens, cylinders d/H=15/30cm. Each cylinder was equipped with three strain gages for obtaining the strains. As it is presented in literature and can be seen in the figures above, steel fibres did not have a big influence on the Modulus of Elasticity.

In the following Figure 7.21, comparison between the stress – strain relationship at the age of 40 and 400 days is presented for all types of concrete. It can be noticed that, in all three types of concrete, there is a small difference in the stress - strain curves due to the increase of the strength of concrete with time. Although not much significant, when the behaviour at the age of 40 and 400 days is compared, it seems that, in the case of C30/37, there is more difference in the inclination of the curve than in the case of concretes C30/37 FL 1.5/1.5 and C30/37 FL 2.5/2.0.

Figure 7.21: Comparison between the stress – strain relationship in compression at the age of 40 versus 400 days for the three concrete types
The same comparison, but with normalized stress, is presented in Figure 7.22.

![Figure 7.22: Comparison between the normalized stress – strain relationship in compression at the age of 40 versus 400 days for the three concrete types](image)

In Figure 7.23 and Figure 7.24, a comparison of the experimental stress – strain relationship in compression tested at 40 and 400 days according to the proposal given in Eurocode 2, is presented. Only the ascending branch of the curve according to EC2 is presented.

It can be noticed that the obtained Elasticity Moduli for the three types of concrete are smaller than the one presented in EC2.

It is well known that the Modulus of Elasticity of concrete depends on the Modulus of Elasticity of its components, especially the aggregate. The values presented in EC2 are general values and are valid for quartzite aggregates. For limestone and sandstone aggregates, which were used in the framework of this research, the given general values of the Modulus of Elasticity should be reduced by 10% and 30%, respectively [13].

The total amount of aggregate used for the preparation of the three types of concrete in this research, consisted of 50% sandstone and 50% limestone. Therefore, the obtained Modulus of Elasticity was smaller than the proposed values for general use for about 20%. For concrete grade C30/37, the poposed value in EC2 is 33000MPa. The expected Modulus of Elasticity for the combination of the used aggregates was $E_{cm} = 0.8 \times 33000 = 26400 \text{ MPa}$.

Therefore, it can be concluded and noticed from the subsequent figures that good agreement was found in all cases.
Figure 7.23: Comparison between the experimental stress – strain relationship in compression at the age of 40 days and Eurocode 2
Figure 7.24: Comparison between the experimental stress – strain relationship in compression at the age of 400 days and Eurocode 2
Each time-dependent deformation property was measured on three control prism specimens with dimensions 12x12x36cm for each type of concrete. The strains were measured on four sides of each prism, which means that, for each type of concrete, the results presented in the subsequent tables and figures represent the mean value obtained at 12 measurement points.

Drying shrinkage was measured beginning with the opening of the moulds of the control specimens to the age of concrete of 400 days. Drying shrinkage was also measured on the compression creep control specimens until the moment of application of the load at the age of 40 days. The instantaneous strain was registered at the moment of application of the load, and afterwards, to the age of concrete of 400 days, creep strains were measured.

Table 7.5 shows the results for the time-dependent properties of each type of concrete. The results for the creep, except through the measured strains, are also presented through the creep coefficient, which is the ratio between the creep strain at 400 days and the instantaneous strain at 40 days. In this way, the creep coefficient at the age of concrete of 400 days is obtained. The total strain at the age of 400 days is also presented in the same table together with the percentual participation of each strain component in the total strain. If the percentual participation is observed, it can be concluded that there is not much difference between the three types of concrete. In the total strain of the three types of concrete, drying shrinkage participates with 53-55.9%, while instantaneous strain and creep strain participate with 17-18.8% and 26.1-28.2%, respectively.

Table 7.5: Time-dependent deformation properties at the age of 40 and 400 days of C30/37, C30/37 FL 1.5/1.5 and C30/37 FL 2.5/2.0

<table>
<thead>
<tr>
<th>Time-dependent properties</th>
<th>Age t(days)</th>
<th>C30/37</th>
<th>%</th>
<th>C30/37 FL1.5/1.5</th>
<th>%</th>
<th>C30/37 FL2.5/2.0</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drying shrinkage $\varepsilon_{ds}$ [$10^{-6}$]μs</td>
<td>40</td>
<td>441.3</td>
<td>/</td>
<td>405.3</td>
<td>/</td>
<td>462.7</td>
<td>/</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>808.0</td>
<td>53</td>
<td>805.0</td>
<td>56.2</td>
<td>794.9</td>
<td>55.9</td>
</tr>
<tr>
<td>Instantaneous strain $\varepsilon_e$ [$10^{-6}$]μs</td>
<td>40</td>
<td>286.3</td>
<td>18.8</td>
<td>254.0</td>
<td>17.7</td>
<td>241.3</td>
<td>17.0</td>
</tr>
<tr>
<td>Creep strain $\varepsilon_{cc}$ [$10^{-6}$]μs</td>
<td>400</td>
<td>429.7</td>
<td>28.2</td>
<td>374.7</td>
<td>26.1</td>
<td>385.0</td>
<td>27.1</td>
</tr>
<tr>
<td>Total strain $\varepsilon=\varepsilon_{ds400}+\varepsilon_{e}+\varepsilon_{cc}$ [$10^{-6}$]μs</td>
<td>400</td>
<td>1524.0</td>
<td>100</td>
<td>1433.7</td>
<td>100</td>
<td>1421.2</td>
<td>100</td>
</tr>
<tr>
<td>Creep coefficient $\varphi(t,t_0)=\varepsilon_{cc}/\varepsilon_e$</td>
<td>400</td>
<td>1.501</td>
<td>/</td>
<td>1.475</td>
<td>/</td>
<td>1.595</td>
<td>/</td>
</tr>
</tbody>
</table>

The decrease of the time-dependent deformation properties in concrete types C30/37 FL 1.5/1.5 and C30/37 FL 2.5/2.0 is presented in Table 7.6.

Table 7.6: Decrease of time-dependent deformation properties of C30/37 FL 1.5/1.5 and C30/37 FL 2.5/2.0

<table>
<thead>
<tr>
<th>Time-dependent properties</th>
<th>Age t(days)</th>
<th>C30/37</th>
<th>C30/37 FL1.5/1.5</th>
<th>C30/37 FL2.5/2.0</th>
<th>decr. %</th>
<th>decr. %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drying shrinkage $\varepsilon_{ds}$ [$10^{-6}$]μs</td>
<td>400</td>
<td>808.0</td>
<td>805.0</td>
<td>0.4</td>
<td>794.9</td>
<td>1.6</td>
</tr>
<tr>
<td>Instantaneous strain $\varepsilon_e$ [$10^{-6}$]μs</td>
<td>40</td>
<td>286.3</td>
<td>254.0</td>
<td>11.3</td>
<td>241.3</td>
<td>15.7</td>
</tr>
<tr>
<td>Creep strain $\varepsilon_{cc}$ [$10^{-6}$]μs</td>
<td>400</td>
<td>429.7</td>
<td>374.7</td>
<td>12.8</td>
<td>385.0</td>
<td>10.4</td>
</tr>
<tr>
<td>Total strain $\varepsilon=\varepsilon_{ds400}+\varepsilon_{e}+\varepsilon_{cc}$ [$10^{-6}$]μs</td>
<td>400</td>
<td>1524.0</td>
<td>1433.7</td>
<td>5.9</td>
<td>1421.2</td>
<td>6.8</td>
</tr>
</tbody>
</table>
The total strain, which is composed from the drying shrinkage, instantaneous and creep strain, in the period from demoulding up to the age of concrete of 400 days, is presented in Figure 7.25 by full lines, while drying shrinkage is presented by dotted lines.

Figure 7.25: Total strain (drying shrinkage, instantaneous and creep) for the three concrete types

Figure 7.26 and Figure 7.27 show the drying shrinkage and the normalized drying shrinkage, respectively. As it can be seen from the figures, and presented earlier in Table 7.6, the fibers did not have big influence on the drying shrinkage strain. In the case of the concrete types C30/37 FL 1.5/1.5 and C30/37 FL 2.5/2.0, drying shrinkage strain decreased for 0.4 and 1.6%. According to the already mentioned theory in Chapter 4.1.3, the decrease was 5.6 and 11.1%:

- For C30/37 FL 1.5/1.5: $\varepsilon_{ds} = \varepsilon_{os} \left(1 - 2.45 \mu \nu, \frac{l}{d}\right) = \varepsilon_{os} \left(1 - 2.45 \cdot 0.12 \cdot \frac{0.38}{100} \cdot 50\right) = 0.944 \varepsilon_{os}$
- For C30/37 FL 2.5/2.0: $\varepsilon_{ds} = \varepsilon_{os} \left(1 - 2.45 \mu \nu, \frac{l}{d}\right) = \varepsilon_{os} \left(1 - 2.45 \cdot 0.12 \cdot \frac{0.76}{100} \cdot 50\right) = 0.888 \varepsilon_{os}$
Figure 7.27: Normalized drying shrinkage strain for the three concrete types

In Figure 7.28 and Figure 7.29, the creep and normalized creep strain are presented, respectively. The instantaneous strain, which is the reason for the appearance of creep strain, is also presented in these figures. As it can be seen from the figures, and presented earlier in Table 7.6, the fibers have an influence on the creep strain. In the case of the concrete types C30/37 FL 1.5/1.5 and C30/37 FL 2.5/2.0, instantaneous strain decreased for 11.3 and 15.7%, while creep strain decreased for 12.8 and 10.4%. According to the already mentioned theory in Chapter 4.1.2, the decrease was 4.5 and 8.9%:

- For C30/37 FL 1.5/1.5: $\varepsilon_{\text{fc}} = \varepsilon_{\text{oc}} \left(1 - 1.96 \mu L f / d \right) = \varepsilon_{\text{oc}} \left(1 - 1.96 \cdot 0.12 \cdot \frac{0.38}{100} \cdot 50 \right) = 0.955 \varepsilon_{\text{oc}}$
- For C30/37 FL 2.5/2.0: $\varepsilon_{\text{fc}} = \varepsilon_{\text{oc}} \left(1 - 1.96 \mu L f / d \right) = \varepsilon_{\text{oc}} \left(1 - 1.96 \cdot 0.12 \cdot \frac{0.76}{100} \cdot 50 \right) = 0.911 \varepsilon_{\text{oc}}$
To obtain the mechanical properties of the used reinforcement in this research, uniaxial tensile test was performed on 10 control steel bars from each diameter. The results are presented in Table 7.7, while the stress-strain relationship is shown in Figure 7.30.

<table>
<thead>
<tr>
<th>Type of reinforcement</th>
<th>Diameter Ø (mm)</th>
<th>Yield strength $f_y$ (MPa)</th>
<th>Tensile strength $f_m$ (MPa)</th>
<th>Modulus of elasticity $E_s$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RA 400/500-2</td>
<td>8</td>
<td>540</td>
<td>596</td>
<td>200000</td>
</tr>
<tr>
<td>RA 400/500-2</td>
<td>10</td>
<td>538</td>
<td>648</td>
<td>200000</td>
</tr>
</tbody>
</table>

Figure 7.29: Normalized creep strain for the three concrete types

Figure 7.30: Stress – strain relationship of reinforcement
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7.7 Analysis of Results from Long-Term Loading

As already presented in Table 7.2, the beams from group 3 and 4 from each series representing different types of concrete (A represents C30/37 without fibers, B represents C30/37 FL 1.5/1.5 with 30kg/m³ fibers and C represents C30/37 FL 2.5/2.0 with 60kg/m³ fibers) were subjected to a long-term load. The beams belonging to group 3 were pre-cracked with the service load “g+q” and then subjected to sustained permanent load “g”, while the beams from group 4 were subjected to permanent sustained and repeated variable load “g±q”. Each group consisted of 2 full scale beams, denoted as A₃₁ and A₃₂, B₃₁ and B₃₂, etc. The first letter represented the series, the first index referred to the group, while the second index indicated the number of the beam.

The results from the long-term loading are presented in terms of:
- Time dependent deflections;
- Time-dependent cracks;
- Time-dependent strains.

7.7.1 Time – Dependent Deflections

7.7.1.1 Time – Dependent Deflections of Beams under Permanent Load “g” (group 3)

Time-dependent deflections are first presented in terms of the load-deflection relationship. This relationship shown in Figure 7.31 provides a clear insight into the applied loading history. The beams were first loaded with permanent load “g”. In order to induce cracks and activate the fibers, the beams were pre-cracked with load “g+q” and then the load was returned to the level of the permanent load, which was acting in the period of 360 days. The values presented in Figure 7.31 are the average values obtained for the two beams from each type of concrete.

![Figure 7.31: Load - deflection relationship for the beams from group 3 for the three concrete types](image)

The time-dependent deflections of each beam from each type of concrete are presented in Figure 7.32 – Figure 7.34. Figure 7.35 shows the total time-dependent deflection as an average value obtained for the two beams from each concrete, while Figure 7.36 displays the normalized...
total time-dependent deflection $a/a_{\text{max}}$.

Figure 7.32: Time-dependent deflection for the beams from group 3, A$_{31}$ & A$_{32}$

Figure 7.33: Time-dependent deflection for the beams from group 3, B$_{31}$ & B$_{32}$

Figure 7.34: Time-dependent deflection for the beams from group 3, C$_{31}$ & C$_{32}$
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Figure 7.35: Total time-dependent deflection for the beams from group 3

Figure 7.36: Normalized total deflection $a/a_{\text{max}}$ for the beams from group 3

Long term deflections are presented in Figure 7.37. The long-term/instantaneous deflection ratio at the level of the permanent load, after pre-cracking, is presented in Figure 7.38.

Figure 7.37: Long-term deflection for the beams from group 3
Figure 7.38: Long-term / Instantaneous deflection for the beams from group 3

Table 7.8 shows the instantaneous deflection first at the level of the permanent load “g”, then at the pre-cracking load level “g+q” and finally, at the level of the permanent load “g” after pre-cracking. The total and long-term deflections after 360 days under load “g” are also presented.

<table>
<thead>
<tr>
<th>Concrete type</th>
<th>C30/37</th>
<th>C30/37 FL 1.5/1.5</th>
<th>C30/37 FL 2.5/2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>A_{31}</td>
<td>A_{32}</td>
<td>B_{31}</td>
</tr>
<tr>
<td>$a_0$ (inst.)_g</td>
<td>0.90</td>
<td>0.79</td>
<td>0.92</td>
</tr>
<tr>
<td>- average -</td>
<td>0.845</td>
<td>0.83</td>
<td>0.815</td>
</tr>
<tr>
<td>$a_0$ (inst.)_{g+q}</td>
<td>1.97</td>
<td>2.53</td>
<td>2.01</td>
</tr>
<tr>
<td>- average -</td>
<td>2.25</td>
<td>1.93</td>
<td>1.66</td>
</tr>
<tr>
<td>$a_0$ (after precr.)_g</td>
<td>1.67</td>
<td>2.21</td>
<td>1.68</td>
</tr>
<tr>
<td>- average -</td>
<td>1.94</td>
<td>1.56</td>
<td>1.31</td>
</tr>
<tr>
<td>$a$ (total)</td>
<td>3.40</td>
<td>3.87</td>
<td>3.17</td>
</tr>
<tr>
<td>- average -</td>
<td>3.635</td>
<td>2.99</td>
<td>2.745</td>
</tr>
<tr>
<td>$a$ (long-term)_g</td>
<td>1.73</td>
<td>1.66</td>
<td>1.49</td>
</tr>
<tr>
<td>- average -</td>
<td>1.695</td>
<td>1.43</td>
<td>1.435</td>
</tr>
</tbody>
</table>

From the figures and from Table 7.8, it can be noticed that the instantaneous, total and long-term deflection decreases with the addition of fibers.

The instantaneous deflection after pre-cracking under load “g+q”, at the level of the permanent load “g”, was decreased for 19.6% and 32.5% for the concrete residual strength classes FL 1.5/1.5 and FL 2.5/2.0, respectively.
The total deflection after exposure to the permanent sustained load “g” for 360 days, was decreased for 17.8% and 24.5% for the concrete residual strength classes FL 1.5/1.5 and FL 2.5/2.0, respectively.

The long-term deflection, which is the total minus the instantaneous deflection after pre-cracking, at the level of the permanent load “g”, was decreased for 15.6% and 15.3%, for the concrete residual strength classes FL 1.5/1.5 and FL 2.5/2.0, respectively. It can be noticed that, with the addition of the fibres, the deflections in both concretes were decreased for about the same percentage.

The ratio of long-term/instantaneous deflections is bigger for the concrete type C30/37 FL 2.5/2.0 due to the fact that it exhibits the smallest instantaneous deflection, while its long term deflection is similar to that of the concrete type C30/37 FL 1.5/1.5. On the other hand, the smallest instantaneous deflection of C30/37 FL 2.5/2.0 is a result of the later appearance of cracks and bigger stiffness of the cross section.

7.7.1.2 Time – Dependent Deflections of Beams under Permanent and Repeated Variable Load “g±q” (Group 4)

Time-dependent deflections for group 4 are also presented in terms of a load-deflection relationship. This relationship shown in Figure 7.39 provides an insight into the applied loading history. Due to the daily measurements of the deflections, the graphs have an area shape. Therefore, for the purpose of clarity as to how the deflections are progressing, the time-dependent deflections in the first ten days for the three concrete types are presented in Figure 7.40. The presented values are average values obtained for the two beams from each type of concrete.

The beams were first loaded with permanent load “g” and after the performance of the measurements, they were loaded additionally with the variable load “q”. The variable load “q” was kept for 8 hours and removed in the next 16 hours. This was repeated every day in the period of 360 days. Before and after each application or removal of the variable load, meaning four times daily, all the necessary measurements were performed.
Figure 7.40: Time-dependent deflection for the beams from group 4 in the first 10 days

The time-dependent deflections of each beam constructed of each type of concrete are presented in Figure 7.41 – Figure 7.43.

Figure 7.41: Time-dependent deflection for the beams from group 4, A41 & A42

Figure 7.42: Time-dependent deflection for the beams from group 4, B41 & B42
Figure 7.43: Time-dependent deflection for the beams from group 4, $C_{41}$ & $C_{42}$

Figure 7.44 displays the total time-dependent deflection as an average value obtained for the two beams from each concrete.

Figure 7.44: Total time-dependent deflection for the beams from group 4

Figure 7.45: Long-term deflection for the beams from group 4
The long term deflections at the end of the time period of 360 days with respect to the instantaneous deflection due to load “g+q” are presented in Figure 7.45, while the long-term/instantaneous deflection ratio due to load “g+q” is presented in Figure 7.46.

Figure 7.46: Long-term / Instantaneous deflection for the beams from group 4

Table 7.9: Deflections of the beams from group 4

<table>
<thead>
<tr>
<th>Concrete type</th>
<th>C30/37</th>
<th>C30/37 FL 1.5/1.5</th>
<th>C30/37 FL 2.5/2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>A(_41)</td>
<td>A(_42)</td>
<td>B(_41)</td>
</tr>
<tr>
<td>Deflection (mm)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a_0) (inst.) (g)</td>
<td>0.79</td>
<td>0.80</td>
<td>1.04</td>
</tr>
<tr>
<td>- average -</td>
<td>0.795</td>
<td>0.975</td>
<td>0.74</td>
</tr>
<tr>
<td>(a_0) (inst.) (g+q)</td>
<td>1.79</td>
<td>2.03</td>
<td>1.76</td>
</tr>
<tr>
<td>- average -</td>
<td>1.91</td>
<td>1.765</td>
<td>1.475</td>
</tr>
<tr>
<td>(a) (total)(g)</td>
<td>4.33</td>
<td>4.80</td>
<td>3.75</td>
</tr>
<tr>
<td>- average -</td>
<td>4.565</td>
<td>3.91</td>
<td>3.54</td>
</tr>
<tr>
<td>(a) (total)(g+q)</td>
<td>4.78</td>
<td>5.31</td>
<td>4.14</td>
</tr>
<tr>
<td>- average -</td>
<td>5.045</td>
<td>4.275</td>
<td>3.905</td>
</tr>
<tr>
<td>(a) (long-term)(g)</td>
<td>2.54</td>
<td>2.77</td>
<td>1.99</td>
</tr>
<tr>
<td>- average -</td>
<td>2.655</td>
<td>2.145</td>
<td>2.065</td>
</tr>
<tr>
<td>(a) (long-term)(g+q)</td>
<td>2.99</td>
<td>3.28</td>
<td>2.38</td>
</tr>
<tr>
<td>- average -</td>
<td>3.135</td>
<td>2.51</td>
<td>2.43</td>
</tr>
</tbody>
</table>

From the figures and Table 7.9, it can be noticed that the instantaneous, total and long-term deflections were decreased with the addition of fibers.
The instantaneous deflection at the level of the service load “g+q” was decreased for 7.6% and 22.8% in the case of the concrete residual strength classes FL 1.5/1.5 and FL 2.5/2.0, respectively.

The total deflection after exposure to permanent sustained and repeated variable load “g±q” for 360 days was decreased for 14.4% and 22.5% at the level of the permanent load and 15.3% and 22.6% at the level of the service load, for the residual class FL 1.5/1.5 and FL 2.5/2.0, respectively.

The long-term deflection after 360 days under load, at the level of the permanent load “g”, which is the total deflection minus the instantaneous deflection at the level of the service load “g+q”, was also decreased for 19.2% and 22.2%, respectively.

The long-term deflection after 360 days under load, at the level of the service load “g+q”, which is the total deflection minus the instantaneous deflection at the level of the service load “g+q”, was decreased for 20.0% and 22.5%, respectively.

It can be noticed that both concretes decreased the deflections for about the same percentage at both load levels with the addition of fibres and the increase of residual strength.

Although the total and long-term deflections of reinforced concrete beams were bigger than those of the SFRC beams, no clear conclusion can be made as to the long-term/instantaneous deflection ratio, Figure 7.46. This is due to the fact that C30/37 FL 1.5/1.5 and C30/37 FL 2.5/2.0 exhibit smaller instantaneous deflections, but also smaller long-term deflections.

7.7.1.3 Comparison of Time-Dependent Deflections between Beams from Group 3 & 4

The comparison between the beams from group 3 and 4, which were subjected to different load histories, was very important for finding out the influence of the repeated variable load on the total and long term deflections. The comparison was made at the level of the permanent load “g”, after a period of loading of 360 days.

Figure 7.47 – Figure 7.49 show the comparison between the total deflections of the beams from group 3 and 4 for each concrete taken separately, while Figure 7.50 includes all concrete types in order to provide a better view of the magnitude of the deflections.
Figure 7.48: Comparison between total deflections of the beams from group 3 & 4 (C30/37 FL 1.5/1.5)

Figure 7.49: Comparison between total deflections of the beams from group 3 & 4 (C30/37 FL 2.5/2.0)

Figure 7.50: Comparison between total deflections of the beams from group 3 & 4 (C30/37 FL 1.5/1.5, C30/37 FL 2.5/2.0, C30/37 FL 1.5/1.5, C30/37 FL 2.5/2.0)

In Figure 7.51 – Figure 7.53, a comparison between the long-term deflections of the beams...
from group 3 and 4 is made for each concrete, while Fig. 7.54 refers to all concrete types.

Figure 7.51: Comparison between long-term deflections of the beams from group 3 & 4 (C30/37)

Figure 7.52: Comparison between long-term deflections of the beams from group 3 & 4 (C30/37 FL 1.5/1.5)

Figure 7.53: Comparison between long-term deflections of the beams from group 3 & 4 (C30/37 FL 2.5/2.0)
Table 7.10 outlines the total and long term deflections of the beams from group 3 and 4. The comparison is made between the different loading histories of each concrete type at the level of the permanent load "g", at the end of the time period of 360 days under load.

<table>
<thead>
<tr>
<th>Concrete type</th>
<th>C30/37</th>
<th>C30/37 FL 1.5/1.5</th>
<th>C30/37 FL 2.5/2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>a (total)(_g) - group 3</td>
<td>3.635</td>
<td>2.99</td>
<td>2.745</td>
</tr>
<tr>
<td>a (total)(_g) - group 4</td>
<td>4.565</td>
<td>3.91</td>
<td>3.54</td>
</tr>
<tr>
<td>ratio: group 4 / group 3</td>
<td>1.26</td>
<td>1.31</td>
<td>1.29</td>
</tr>
<tr>
<td>a (long-term)(_g) - group 3</td>
<td>1.695</td>
<td>1.43</td>
<td>1.435</td>
</tr>
<tr>
<td>a (long-term)(_g) - group 4</td>
<td>2.655</td>
<td>2.145</td>
<td>2.065</td>
</tr>
<tr>
<td>ratio: group 4 / group 3</td>
<td>1.57</td>
<td>1.50</td>
<td>1.44</td>
</tr>
</tbody>
</table>

The total deflections are first compared and the corresponding ratios are presented. They are in the range of 1.26 to 1.31. Due to the different instantaneous deflections, which are included in the total deflections, they do not differ much. If the ratios for the long term deflections, namely, 1.57 for C30/37, 1.50 for C30/37 FL 1.5/1.5 and 1.44 for C30/37 FL 1.5/1.5 are compared, it can be concluded that the ratios for the SFR concretes are decreased for 4.5% and 7.3%, respectively.
7.7.2 Time – Dependent Cracks
Although all cracks that appeared during the application of the load or in the period of observation of 360 days were registered, the crack widths were measured only in the middle third of the beams, i.e. in the part of the beams with constant bending moment.

7.7.2.1 Time – Dependent Crack Width of Beams under Permanent Load “g” (Group 3)
The beams from group 3 were first loaded with the permanent load “g”. In order to induce cracks and activate the fibers, the beams were pre-cracked with load “g+q” and than the load was returned to the level of the permanent load which was acting in the period of 360 days. The time-dependent crack-width of each beam is presented in Figure 7.55 – Figure 7.57.

![Figure 7.55: Time-dependent crack widths for the beams from group 3, A_{31} & A_{32}](image)

![Figure 7.56: Time-dependent crack widths for the beams from group 3, B_{31} & B_{32}](image)
Figure 7.57: Time-dependent crack widths for the beams from group 3, C\textsubscript{31} & C\textsubscript{32}

Figure 7.58 shows the time-dependent crack-width as an average value obtained for the two beams.

As predicted, the cracks in all beams appeared between the level of the permanent and the service load. At the reinforced concrete beams (C30/37), A\textsubscript{31} and A\textsubscript{32}, 4 and 6 cracks appeared due to the pre-cracking load, respectively. At the steel fibre reinforced concrete beams (C30/37 FL 1.5/1.5), B\textsubscript{31} and B\textsubscript{32}, 2 cracks appeared in each beam, while at the beams from C30/37 FL 2.5/2.0, C\textsubscript{31} and C\textsubscript{32}, 2 and 1 crack appeared. After removal of the variable load \textquoteright q\textquoteright, the crack widths decreased and increased thereafter up to the moment of appearance of new cracks. The mentioned new cracks were with a smaller crack width wherefore they decreased the average value of all crack widths. This can also be noticed in the figures referring to the first 20 to 100 days. In total, at beams A\textsubscript{31} and A\textsubscript{32}, 11 and 7 cracks developed, at beams B\textsubscript{31} and B\textsubscript{32}, 6 and 3 cracks developed, while at beams C\textsubscript{31} and C\textsubscript{32}, 4 and 8 cracks developed, respectively. After the formation of all cracks, they exhibited only an increase of their width.
Table 7.11: Crack widths of the beams from group 3

<table>
<thead>
<tr>
<th>Concrete type</th>
<th>C30/37</th>
<th>C30/37 FL 1.5/1.5</th>
<th>C30/37 FL 2.5/2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>A31</td>
<td>A32</td>
<td>B31</td>
</tr>
<tr>
<td>Crack width (mm)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w_0 ) (inst.) ( g )</td>
<td>0.090</td>
<td>0.127</td>
<td>0.120</td>
</tr>
<tr>
<td>- average -</td>
<td>0.108</td>
<td>0.130</td>
<td></td>
</tr>
<tr>
<td>( w_0 ) (after precr.) ( g )</td>
<td>0.070</td>
<td>0.100</td>
<td>0.100</td>
</tr>
<tr>
<td>- average -</td>
<td>0.085</td>
<td>0.100</td>
<td></td>
</tr>
<tr>
<td>( w ) (total)</td>
<td>0.096</td>
<td>0.169</td>
<td>0.110</td>
</tr>
<tr>
<td>- average -</td>
<td>0.132</td>
<td>0.135</td>
<td></td>
</tr>
<tr>
<td>( w ) (long-term) ( g )</td>
<td>0.026</td>
<td>0.069</td>
<td>0.010</td>
</tr>
<tr>
<td>- average -</td>
<td>0.048</td>
<td>0.035</td>
<td></td>
</tr>
</tbody>
</table>

The results on the crack widths (Table 7.11) are presented in terms of instantaneous crack width at the moment of pre-cracking and after pre-cracking, total crack width under permanent load "\( g \)" and long-term crack width measured after removal of the variable load at the end of the period of 360 days under permanent load "\( g \)".

From the figures and from Table 7.11, it can be noticed that the instantaneous crack widths at the moment of pre-cracking and after pre-cracking in the case of C30/37 FL 1.5/1.5 are bigger than those obtained for C30/37, while in the case of C30/37 FL 2.5/2.0, they are significantly smaller. However, the total crack width obtained for C30/37 and C30/37 FL 1.5/1.5 is almost the same, while that obtained for C30/37 FL 2.5/2.0 is reduced for 31.8%.

On the other hand, the long-term crack widths referring to C30/37 and C30/37 FL 2.5/2.0 are similar, while those observed in the case of C30/37 FL 1.5/1.5 are decreased for 27.1%. The long-term crack width of C30/37 FL 2.5/2.0 is of the same order of magnitude as that of C30/37 due to the fact that the instantaneous deflection of C30/37 FL 2.5/2.0 is much smaller and the cracks due to loading opened up later.

However, having in mind the complexity of the process of cracking, the randomly oriented fibres and the low level of stress from the permanent load, which is about 20% of the concrete strength, it can be concluded that steel fibres decrease the crack widths.

After exposure to long-term loading for 360 days, ultimate load tests were performed on all beams. The final crack pattern distributions are presented in Figure 7.59 – Figure 7.61. In these figures, one can distinguish between the time dependent cracks, which are thicker, and cracks that appeared later during the ULT.

A summary of crack spacing for all 24 beams is presented in Table 7.13.
Figure 7.59: Crack pattern distribution after ULT for the beams from group 3, A31 & A32

Figure 7.60: Crack pattern distribution after ULT for the beams from group 3, B31 & B32
7.7.2.2 Time – Dependent Crack Width of Beams under Permanent and Repeated Variable Load “g+q” (Group 4)

The beams from group 4 were first loaded with permanent load "g". At this load level, there was no appearance of cracks. Afterwards, the beams were loaded additionally with variable load "q". The variable load “q” was kept for 8 hours and removed in the next 16 hours. This was repeated every day in the period of 360 days. At certain time intervals, before and after application or removal of the variable load, measurements of the crack widths were performed by a crack microscope.

The time-dependent crack-width of each beam from each type of concrete is presented in Figure 7.62 – Figure 7.64. In Figure 7.65, the time-dependent crack-width is presented as an average value obtained for two beams. The dashed lines represent the crack widths at the level of the permanent load “g”, while the full lines represent the crack widths at the level of the service load “g+q”.

Figure 7.61: Crack pattern distribution after ULT for the beams from group 3, C31 & C32
### Figure 7.62: Time-dependent crack widths for the beams from group 4, A41 & A42

![Time-dependent crack widths for group A41 & A42](attachment:image1)

- Concrete type: C30/37
- A41 (g±q)
- A41 (g)
- A42 (g±q)
- A42 (g)

### Figure 7.63: Time-dependent crack widths for the beams from group 4, B41 & B42

![Time-dependent crack widths for group B41 & B42](attachment:image2)

- Concrete type: C30/37 FL 1.5/1.5
- B41 (g±q)
- B41 (g)
- B42 (g±q)
- B42 (g)

### Figure 7.64: Time-dependent crack widths for the beams from group 4, C41 & C42

![Time-dependent crack widths for group C41 & C42](attachment:image3)

- Concrete type: C30/37 FL 2.5/2.0
- C41 (g±q)
- C41 (g)
- C42 (g±q)
- C42 (g)
Figure 7.65: Time-dependent crack widths for the beams from group 4

Table 7.12: Crack widths of the beams from group 4

<table>
<thead>
<tr>
<th>Concrete type</th>
<th>C30/37</th>
<th>C30/37 FL 1.5/1.5</th>
<th>C30/37 FL 2.5/2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>A_{41}</td>
<td>A_{42}</td>
<td>B_{41}</td>
</tr>
<tr>
<td>w_{0} (inst.)_{g+q}</td>
<td>0.105</td>
<td>0.110</td>
<td>0.072</td>
</tr>
<tr>
<td>- average -</td>
<td>0.108</td>
<td>0.089</td>
<td></td>
</tr>
<tr>
<td>w (total)_{g+q}</td>
<td>0.218</td>
<td>0.235</td>
<td>0.170</td>
</tr>
<tr>
<td>- average -</td>
<td>0.226</td>
<td>0.160</td>
<td></td>
</tr>
<tr>
<td>w (total)_{g}</td>
<td>0.182</td>
<td>0.195</td>
<td>0.145</td>
</tr>
<tr>
<td>- average -</td>
<td>0.189</td>
<td>0.136</td>
<td></td>
</tr>
<tr>
<td>w (long-term)_{g+q}</td>
<td>0.113</td>
<td>0.125</td>
<td>0.098</td>
</tr>
<tr>
<td>- average -</td>
<td>0.119</td>
<td>0.071</td>
<td></td>
</tr>
<tr>
<td>w (long-term)_{g}</td>
<td>0.077</td>
<td>0.085</td>
<td>0.073</td>
</tr>
<tr>
<td>- average -</td>
<td>0.081</td>
<td>0.047</td>
<td></td>
</tr>
</tbody>
</table>

The presented figures and Table 7.12 point to a much clearer situation than that in the case of group 3. This is due to the higher load level in the time intervals of 8 hours each day in the case of group 4.

The instantenous crack widths at the level of the service load for concrete type C30/37 FL 1.5/1.5 are 17.6% smaller than those for C30/37, while for C30/37 FL 2.5/2.0, they are 58.4% smaller than those for C30/37.

The total crack widths at the level of the service load, when compared to C30/37, were decreases for 29.2% and 53.1% for C30/37 FL 1.5/1.5 and C30/37 FL 2.5/2.0, respectively. At the
level of the permanent load, a similar decrease was obtained, namely, 28.1% and 53.4% for C30/37 FL 1.5/1.5 and C30/37 FL 2.5/2.0, respectively.

The long-term crack widths at the level of the service load, when compared to C30/37, were decreased for 40.3% and 47.9% in the case of C30/37 FL 1.5/1.5 and C30/37 FL 2.5/2.0, respectively. At the level of the permanent load, a similar decrease was obtained, i.e., 42.0% and 46.9% for C30/37 FL 1.5/1.5 and C30/37 FL 2.5/2.0, respectively.

As for the beams from group 3, ultimate load testing was performed also for the beams from group 4. The final crack pattern distributions are presented in Figure 7.66 – Figure 7.68. In these figures, one can distinguish between the time dependent cracks, which are thicker, and cracks that appeared later during the ULT.

A summary of the crack spacing for all 24 beams is presented in Table 7.13

Figure 7.66: Crack pattern distribution after ULT for the beams from group 4, A41 & A42
Figure 7.67: Crack pattern distribution after ULT for the beams from group 4, B41 & B42

Figure 7.68: Crack pattern distribution after ULT for the beams from group 4, C41 & C42
7.7.3 Crack Spacing of Full Scale Beams

Ultimate load tests were carried out on the beams from group 3 and 4, which were subjected to different load histories for 360 days. The beams from group 1 and 2 were also subjected to ULT. The results obtained for all 24 beams from the four groups are presented in Table 7.13.

The measured average crack spacing was 14.31 cm, 11.86 cm and 11.99 cm for C30/37, C30/37 FL 1.5/1.5 and C30/37 FL 2.5/2.0, respectively. The decrease in the crack spacing was 17.12% in the case of C30/37 FL 1.5/1.5 and 16.21% in the case of C30/37 FL 2.5/2.0. This means that the addition of fibers decreases the crack spacing, but the amount of the steel fibers (30 or 60 kg/m³) does not have influence on the crack spacing.

RILEM TC 162, proposed an equation, which is a modification of the equation in Eurocode 2 (1991) for fibre concretes:

\[
s_{m} = 50 + 0.25k_{f}a_{fr} \cdot \frac{50}{L_{f}/d_{f}} \quad [25]
\]

(7.1)

Actually, Eurocode 2 (1991) gives only the part of the equation in the first parenthesis, while the part in the second parenthesis is the modification of RILEM TC 162. With this modification, the steel fibers are introduced by taking into account the aspect ratio of the fibers \((L_f/d_f)\) and a coefficient of 50 obtained by a regression analysis of experimental results. The other coefficients are the same as for reinforced concrete: \(k_1\) takes into account the bond properties of the reinforcing steel (0.8 for ribbed and 1.6 for smooth bars), \(k_2\) takes into account the form of the strain distribution (0.5 for bending and 1.0 tension), \(\rho_r\) is the effective reinforcement ratio and is a ratio between the reinforcement section within the effective tensile zone and the concrete area surrounding the tensile reinforcement up to a height of 2.5 times the distance of the centre of the reinforcement to the most stretched fibre.

The crack spacing calculated according to Eq. 7.1 is presented in Table 7.13. In this way, the RILEM proposal is confirmed in the sense that the fibre dosages do not influence much the crack spacing, but the aspect ratio. If the aspect ratio is the same, as it is in this case (50), the crack spacing will be the same. However, on the other hand, with the proposed equation, which is valid for aspect ratios \(L_f/d_f \geq 50\), for an aspect ratio of 50, there should be no difference in the crack spacing. Therefore, for an aspect ratio of 50, a different equation should be proposed.
7.7.4 Time – Dependent Strains

Time-dependent strains on each beam that was subjected to long-term loading were measured at 5 points along the middle section of the beam. The measurements were performed at the level of the tensile and compressive reinforcement, at the concrete compression edge, as well as at other two points in the middle of the beam depth.

7.7.4.1 Time – Dependent Strains of Beams under Permanent Load “g” (Group 3)

The time-dependent strains of each beam from each type of concrete are presented in Table 7.14.

![Table 7.13: Crack spacing of all 24 full scale beams]

<table>
<thead>
<tr>
<th>Type of Concrete</th>
<th>Beam</th>
<th>Crack spacing $s_{m}$ (cm)</th>
<th>Average crack spacing $s_{m}$ (cm)</th>
<th>Crack spacing EC2&amp;RILEM $s_{m}$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C30/37</td>
<td>A11</td>
<td>13.10</td>
<td>13.90</td>
<td>14.31</td>
</tr>
<tr>
<td></td>
<td>A12</td>
<td>14.70</td>
<td></td>
<td>13.36</td>
</tr>
<tr>
<td></td>
<td>A21</td>
<td>15.12</td>
<td>13.40</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A22</td>
<td>11.68</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>A31</td>
<td>12.21</td>
<td>13.19</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A32</td>
<td>14.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>A41</td>
<td>18.88</td>
<td>16.74</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A42</td>
<td>14.60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C30/37 FL 1.5/1.5</td>
<td>B11</td>
<td>11.99</td>
<td>12.83</td>
<td>11.86</td>
</tr>
<tr>
<td></td>
<td>B12</td>
<td>13.68</td>
<td></td>
<td>13.36</td>
</tr>
<tr>
<td></td>
<td>B21</td>
<td>10.91</td>
<td>11.83</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B22</td>
<td>12.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>B31</td>
<td>10.08</td>
<td>10.67</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B32</td>
<td>11.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>B41</td>
<td>10.67</td>
<td>12.11</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B42</td>
<td>13.56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C30/37 FL 2.5/2.0</td>
<td>C11</td>
<td>9.14</td>
<td>11.82</td>
<td>11.99</td>
</tr>
<tr>
<td></td>
<td>C12</td>
<td>14.50</td>
<td></td>
<td>13.36</td>
</tr>
<tr>
<td></td>
<td>C21</td>
<td>10.40</td>
<td>10.88</td>
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<td></td>
<td>C22</td>
<td>11.36</td>
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<td></td>
<td>C31</td>
<td>13.88</td>
<td>13.58</td>
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<td></td>
<td>C32</td>
<td>13.28</td>
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<td></td>
<td>C41</td>
<td>12.72</td>
<td>11.68</td>
<td></td>
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<tr>
<td></td>
<td>C42</td>
<td>10.65</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 7.14: Time-dependent strain distribution for the beams from group 3 after 1, 20 and 360 days under sustained load $F_g$

<table>
<thead>
<tr>
<th>Type of Concrete</th>
<th>Beam</th>
<th>Load</th>
<th>n [days under load]</th>
<th>Measured strains</th>
</tr>
</thead>
<tbody>
<tr>
<td>C30/37</td>
<td>A31</td>
<td>$F_{g+q}$</td>
<td>$n=1$</td>
<td>$\varepsilon_s [%]$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_g$</td>
<td>$n=1$</td>
<td>0.560</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_g$</td>
<td>$n=20$</td>
<td>0.534</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_g$</td>
<td>$n=360$</td>
<td>0.518</td>
</tr>
<tr>
<td></td>
<td>A32</td>
<td>$F_{g+q}$</td>
<td>$n=1$</td>
<td>0.866</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_g$</td>
<td>$n=1$</td>
<td>0.712</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_g$</td>
<td>$n=20$</td>
<td>0.738</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_g$</td>
<td>$n=360$</td>
<td>0.704</td>
</tr>
<tr>
<td>C30/37 FL 1.5/1.5</td>
<td>B31</td>
<td>$F_{g+q}$</td>
<td>$n=1$</td>
<td>0.690</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_g$</td>
<td>$n=1$</td>
<td>0.590</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_g$</td>
<td>$n=20$</td>
<td>0.576</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_g$</td>
<td>$n=360$</td>
<td>0.508</td>
</tr>
<tr>
<td>C30/37 FL 2.5/2.0</td>
<td>B32</td>
<td>$F_{g+q}$</td>
<td>$n=1$</td>
<td>0.680</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_g$</td>
<td>$n=1$</td>
<td>0.564</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_g$</td>
<td>$n=20$</td>
<td>0.544</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_g$</td>
<td>$n=360$</td>
<td>0.482</td>
</tr>
<tr>
<td>C30/37 FL 2.5/2.0</td>
<td>C31</td>
<td>$F_{g+q}$</td>
<td>$n=1$</td>
<td>0.448</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_g$</td>
<td>$n=1$</td>
<td>0.366</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_g$</td>
<td>$n=20$</td>
<td>0.350</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_g$</td>
<td>$n=360$</td>
<td>0.326</td>
</tr>
<tr>
<td></td>
<td>C32</td>
<td>$F_{g+q}$</td>
<td>$n=1$</td>
<td>0.276</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_g$</td>
<td>$n=1$</td>
<td>0.222</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_g$</td>
<td>$n=20$</td>
<td>0.194</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_g$</td>
<td>$n=360$</td>
<td>0.128</td>
</tr>
</tbody>
</table>

The strains in the previous table refer to the moment of pre-cracking (“$g+q$”) and after 1, 20 and 360 days under sustained load “$g$”.

The time-dependent strains of each beam from each type of concrete are presented graphically in Figure 7.69 – Figure 7.74. The height of the beam is presented on the y-axis. In addition to the strains presented in the table, in the subsequent figures, the strains at the level of the sustained permanent load before pre-cracking are also presented.
Figure 7.69: Time-dependent strain distribution for beam A31, concrete type C30/37

Figure 7.70: Time-dependent strain distribution for beam A32, concrete type C30/37

Figure 7.71: Time-dependent strain distribution for beam B31, concrete type C30/37 FL 1.5/1.5
From the previous figures, the increase in the strains at the upper compressive edge of all beams is the most evident one, while at the tensile reinforcement, there is a more or less stabilized situation with a decrease of strains with time.

To get a much clearer insight into the influence of the steel fibers on the strain distribution, Figure 7.75 – Figure 7.78 display the total time-dependent strains as an average value obtained for two beams from each concrete type. Figure 7.75 presents the strain distribution at the moment of pre-cracking under the service load "g+q", while Figures 7.76 – Figure 7.78 show the time-
dependent strain distribution after 1, 20 and 360 days under sustained load “g”.

Figure 7.75: Time-dependent strain distribution for the beams from group 3 for the three concrete types at pre-cracking load $F_{g+q}$

Figure 7.76: Time-dependent strain distribution for the beams from group 3 for the three concrete types after 1 day under sustained load $F_g$

Figure 7.77: Time-dependent strain distribution for the beams from group 3 for the three concrete types after 20 days under sustained load $F_g$
Figure 7.78: Time-dependent strain distribution for the beams from group 3 for the three concrete types after 360 days under sustained load $F_g$

The same results presented in the previous figures are summarized in Table 7.15.

Table 7.15: Time-dependent strain distribution for the beams from group 3 after 1, 20 and 360 days under sustained load $F_g$ as mean values

<table>
<thead>
<tr>
<th>Type of Concrete</th>
<th>Load</th>
<th>$n$ [days under load]</th>
<th>Measured strains</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Tensile reinforcement $\varepsilon_s [%]$</td>
</tr>
<tr>
<td>C30/37</td>
<td>$F_g+q$</td>
<td>n=1</td>
<td>0.713</td>
</tr>
<tr>
<td></td>
<td>$F_g$</td>
<td>n=1</td>
<td>0.628</td>
</tr>
<tr>
<td></td>
<td>$F_g$</td>
<td>n=20</td>
<td>0.636</td>
</tr>
<tr>
<td></td>
<td>$F_g$</td>
<td>n=360</td>
<td>0.611</td>
</tr>
<tr>
<td>C30/37 FL 1.5/1.5</td>
<td>$F_g+q$</td>
<td>n=1</td>
<td>0.685</td>
</tr>
<tr>
<td></td>
<td>$F_g$</td>
<td>n=1</td>
<td>0.577</td>
</tr>
<tr>
<td></td>
<td>$F_g$</td>
<td>n=20</td>
<td>0.560</td>
</tr>
<tr>
<td></td>
<td>$F_g$</td>
<td>n=360</td>
<td>0.495</td>
</tr>
<tr>
<td>C30/37 FL 2.5/2.0</td>
<td>$F_g+q$</td>
<td>n=1</td>
<td>0.362</td>
</tr>
<tr>
<td></td>
<td>$F_g$</td>
<td>n=1</td>
<td>0.294</td>
</tr>
<tr>
<td></td>
<td>$F_g$</td>
<td>n=20</td>
<td>0.272</td>
</tr>
<tr>
<td></td>
<td>$F_g$</td>
<td>n=360</td>
<td>0.227</td>
</tr>
</tbody>
</table>

At the concrete compression edge, at the moment of pre-cracking, concrete type C30/37 FL 1.5/1.5 exhibited a strain that was bigger than that of C30/37 for 17.1%, while C30/37 FL 2.5/2.0 showed a 15.8% smaller strain. The same percents remained after removal of load “q”. The compressive strains increased with time and, after 360 days under sustained permanent load, they were bigger for 7.2% in the case of C30/37 FL 1.5/1.5 and smaller for 3.4% in the case of C30/37 FL 2.5/2.0 than those of C30/37. At the level of the permanent load “g”, the increase from the moment after pre-cracking, i.e., $n=1$ day to $n=360$ days was 3.3 times for C30/37, 2.97 times for C30/37 FL 1.5/1.5 and 3.71 times for C30/37 FL 2.5/2.0. It can therefore be concluded that...
steel fibres do not have any significant influence on the concrete compressive time-dependent strains.

At the level of the tensile reinforcement, the development of strains was different than that at the compressive edge. At the moment of pre-cracking, concrete type C30/37 FL 1.5/1.5 exhibited a 4% smaller strain than that of C30/37, while C30/37 FL 2.5/2.0 showed a 49.2% smaller strain. After removal of the variable load, the beams were subjected only to permanent sustained load for 360 days. In this time period, there was only decreasing of the strain in each beam and in each type of concrete. After 360 days under load, the final tensile strain in the case of concrete type C30/37 FL 1.5/1.5 was 19% smaller than that of C30/37, while in the case of C30/37 FL 2.5/2.0, it was 62.8% smaller than that of C30/37. From these results, it can be concluded that steel fibres have a beneficial effect on the time – dependent strain distribution.

7.7.4.2 Time – Dependent Strains of Beams under Permanent and Repeated Variable Load “g+q” (Group 4)

Time – dependent strains of each beam from group 4 and each type of concrete are presented in Table 7.16 and graphically in Figure 7.79 – Figure 7.84. The beams from this group were subjected to the permanent sustained load daily, while the variable load acted for 8 hours in the course of the day and was removed for the rest 16 hours. Therefore, the strains are presented at the level of the sustained permanent load “g” and at the level of the service load “g+q” after 1, 20 and 360 days under this load history.

Figure 7.79: Time-dependent strain distribution for beam A41, concrete type C30/37

Figure 7.80: Time-dependent strain distribution for beam A42, concrete type C30/37
Table 7.16: Time dependent strain distribution for the beams from group 4 after 1 and 360 days under sustained and repeated variable load $F_{g+q}$

<table>
<thead>
<tr>
<th>Type of Concrete</th>
<th>Beam</th>
<th>Load</th>
<th>n [days under load]</th>
<th>Measured strains</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Tensile reinforcement $\varepsilon_s[^{\circ}]$</td>
</tr>
<tr>
<td>C30/37</td>
<td>A_{41}</td>
<td>$F_g$</td>
<td>n=1</td>
<td>0.104</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_g$</td>
<td>n=360</td>
<td>0.886</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_{g+q}$</td>
<td>n=1</td>
<td>0.768</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_{g+q}$</td>
<td>n=360</td>
<td>1.034</td>
</tr>
<tr>
<td></td>
<td>A_{42}</td>
<td>$F_g$</td>
<td>n=1</td>
<td>0.092</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_g$</td>
<td>n=360</td>
<td>0.664</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_{g+q}$</td>
<td>n=1</td>
<td>0.524</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_{g+q}$</td>
<td>n=360</td>
<td>0.788</td>
</tr>
<tr>
<td>C30/37 FL 1.5/1.5</td>
<td>B_{41}</td>
<td>$F_g$</td>
<td>n=1</td>
<td>0.140</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_{g+q}$</td>
<td>n=1</td>
<td>0.428</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_g$</td>
<td>n=360</td>
<td>0.820</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_{g+q}$</td>
<td>n=360</td>
<td>0.920</td>
</tr>
<tr>
<td></td>
<td>B_{42}</td>
<td>$F_g$</td>
<td>n=1</td>
<td>0.106</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_{g+q}$</td>
<td>n=1</td>
<td>0.566</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_g$</td>
<td>n=360</td>
<td>0.448</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_{g+q}$</td>
<td>n=360</td>
<td>0.560</td>
</tr>
<tr>
<td>C30/37 FL 2.5/2.0</td>
<td>C_{41}</td>
<td>$F_g$</td>
<td>n=1</td>
<td>0.100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_{g+q}$</td>
<td>n=1</td>
<td>0.180</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_g$</td>
<td>n=360</td>
<td>0.280</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_{g+q}$</td>
<td>n=360</td>
<td>0.332</td>
</tr>
<tr>
<td></td>
<td>C_{42}</td>
<td>$F_g$</td>
<td>n=1</td>
<td>0.076</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_{g+q}$</td>
<td>n=1</td>
<td>0.154</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_g$</td>
<td>n=360</td>
<td>0.156</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_{g+q}$</td>
<td>n=360</td>
<td>0.196</td>
</tr>
</tbody>
</table>
Similar to the beams from group 3, the increase of strains at the upper compressive edge of all beams is the most evident one, while due to the higher level of load, at the tensile reinforcement, there is an increase of strain, which is more significant in the case of concrete type C30/37 and C30/37 FL 1.5/1.5 than in the case of C30/37 FL 2.5/2.0.
Figure 7.84: Time-dependent strain distribution for beam C_{42}, concrete type C30/37 FL 2.5/2.0

In Table 7.17 and Figure 7.85 through Figure 7.87, the total time-dependent strains are presented as an average value obtained for two beams from each concrete type. The figures present the time-dependent strain distribution after 1, 20 and 360 days under sustained and repeated variable load "F_{g+q}".

Figure 7.85: Time-dependent strain distribution for the beams from group 4 for the three concrete types after 1 day under sustained and repeated variable load F_{g+q}

Figure 7.86: Time-dependent strain distribution for the beams from group 4 for the three concrete types after 20 days under sustained and repeated variable load F_{g+q}
Figure 7.87: Time-dependent strain distribution for the beams from group 4 for the three concrete types after 360 days under sustained and repeated variable load $F_{g+q}$.

Table 7.17: Time dependent strain distribution for the beams from group 4 after 1 and 360 days under sustained and repeated variable load $F_{g+q}$ as mean values.

<table>
<thead>
<tr>
<th>Type of Concrete</th>
<th>Load</th>
<th>n [days under load]</th>
<th>Measured strains</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Tensile reinforcement $\varepsilon_s[%]$</td>
</tr>
<tr>
<td>C30/37</td>
<td>$F_g$</td>
<td>n=1</td>
<td>0.098</td>
</tr>
<tr>
<td></td>
<td>$F_{g+q}$</td>
<td>n=1</td>
<td>0.646</td>
</tr>
<tr>
<td></td>
<td>$F_g$</td>
<td>n=360</td>
<td>0.775</td>
</tr>
<tr>
<td></td>
<td>$F_{g+q}$</td>
<td>n=360</td>
<td>0.911</td>
</tr>
<tr>
<td>C30/37 FL 1.5/1.5</td>
<td>$F_g$</td>
<td>n=1</td>
<td>0.123</td>
</tr>
<tr>
<td></td>
<td>$F_{g+q}$</td>
<td>n=1</td>
<td>0.497</td>
</tr>
<tr>
<td></td>
<td>$F_g$</td>
<td>n=360</td>
<td>0.634</td>
</tr>
<tr>
<td></td>
<td>$F_{g+q}$</td>
<td>n=360</td>
<td>0.740</td>
</tr>
<tr>
<td>C30/37 FL 2.5/2.0</td>
<td>$F_g$</td>
<td>n=1</td>
<td>0.088</td>
</tr>
<tr>
<td></td>
<td>$F_{g+q}$</td>
<td>n=1</td>
<td>0.167</td>
</tr>
<tr>
<td></td>
<td>$F_g$</td>
<td>n=360</td>
<td>0.218</td>
</tr>
<tr>
<td></td>
<td>$F_{g+q}$</td>
<td>n=360</td>
<td>0.264</td>
</tr>
</tbody>
</table>

At the concrete compression edge, at the level of the permanent load on the first day, in the case of concrete type C30/37 FL 1.5/1.5, the strain is bigger for 12.5% than that of C30/37, while in the case of C30/37 FL 2.5/2.0, there is less than 1% difference in strain. Already at the level of the service load, in the case of concrete type C30/37 FL 1.5/1.5, there is a strain which is smaller for 9.5% than that of C30/37, while in the case of C30/37 FL 2.5/2.0, there is a 35.9% smaller strain. After 360 days under the specific load history, at the level of the permanent load, in the case of concrete type C30/37 FL 1.5/1.5, there is a strain that is bigger for 19.5% than that of C30/37, while in the case of C30/37 FL 2.5/2.0, there is again less than 1% difference in strain. At the level of the service load, in the case of concrete type C30/37 FL 1.5/1.5, there is a strain that
is bigger for 17.8% than that of C30/37, while in the case of C30/37 FL 2.5/2.0, there is 2% smaller strain. The increase of strain at the level of the permanent load “g”, from the first day, n=1, to n=360 days is 8.42, 8.95 and 8.29 times in the case of C30/37, C30/37 FL 1.5/1.5 and C30/37 FL 2.5/2.0, respectively. The increase at the level of the service load “g+q”, from the first day, n=1, to n=360 days is 3.0 times in the case of C30/37, 3.91 in the case of C30/37 FL 1.5/1.5 and 4.59 in the case of C30/37 FL 2.5/2.0. Therefore, similar to group 3, for the case of group 4, it can be concluded that steel fibres do not have any significant influence on the concrete compressive time-dependent strains.

At the level of the tensile reinforcement, the development of strains is different than that at the compressive edge. At the level of the permanent load, on the first day, in the case of concrete type C30/37 FL 1.5/1.5, there is a strain that is bigger for 25.5% than that of C30/37, while in the case of C30/37 FL 2.5/2.0, there is 10% smaller strain. At the level of the service load, in the case of concrete type C30/37 FL 1.5/1.5, there is 23% smaller strain than that of C30/37, while in the case of C30/37 FL 2.5/2.0, there is 74.1% smaller strain. After 360 days under the specific load history, at the level of the permanent load, in the case of concrete type C30/37 FL 1.5/1.5, there is a strain that is smaller for 18.2% than that of C30/37, while in the case of C30/37 FL 2.5/2.0, there is 72.9% smaller strain. At the level of the service load, in the case of concrete type C30/37 FL 1.5/1.5, there is a strain that is smaller for 18.8% than that of C30/37, while in the case of C30/37 FL 2.5/2.0, there is 71% smaller strain. The increase of strain at the level of the permanent load “g”, from the first day, n=1, to n=360 days is 7.91, 5.15 and 2.48 times in the case of C30/37, C30/37 FL 1.5/1.5 and C30/37 FL 2.5/2.0, respectively. The increase at the level of the service load “g+q”, from the first day, n=1, to n=360 days is 1.41 times in the case of C30/37, 1.49 in the case of C30/37 FL 1.5/1.5 and 1.58 in the case of C30/37 FL 2.5/2.0. From these results, it can be concluded again that steel fibres have a beneficial effect on the time-dependent strain distribution. This is due to the fact that they sustain part of the tensile force, which is sustained only by the tensile reinforcement in the case of ordinary reinforced concrete.
CHAPTER 8: ANALYTICAL ANALYSES OF RESULTS FROM EXPERIMENTAL RESEARCH

The analytical analyses of the results from the experimental research were performed in two parts:

- Analytical analysis of time – dependent deformation properties,
- Analytical analysis of time – dependent deflections.

Data on the time – dependent deformation properties were later used to calculate the time-dependent deflections using the Age-Adjusted Effective Modulus Method (AAEMM). The quasi-permanent load procedure and the principle of superposition were used to obtain the factor for the quasi-permanent value of the variable action $\psi_2$.

8.1 Analytical Analysis of Time – Dependent Deformation Properties

8.1.1 Analytical Analysis of Drying Shrinkage

The analytical analysis of time – dependent deformation properties, drying shrinkage and creep, were performed by the B3 model and Fib Model Code 2010. In the beginning, the analyses were done only for the time period considered in this research, which was 400 days.

The B3 model offers the possibility of improvement of the model by its users and updating of its predictions based on short-time measurements. The updating of the drying shrinkage strain was done very efficiently by using the scaling parameter $p_6$.

The experimental and analytical results for the drying shrinkage up to the age of 400 days are presented in Figure 8.1 while the final values of the drying shrinkage strain at 400 days are presented in Table 8.1. It can be noticed that the Fib Model Code 2010 underestimates the drying shrinkage strain for 29%, while the original B3 model underestimation is 11.5%. The obtained scaling parameter in the improved B3 model is $p_6=1.123$. It can be noticed that there is a very good agreement between the experimental results and the improved B3 model.

![Figure 8.1: Experimental and analytical results on drying shrinkage up to age of 400 days](image-url)
Table 8.1: Results from the analytical analysis of drying shrinkage at the age of 400 days

<table>
<thead>
<tr>
<th>Drying shrinkage $\varepsilon_{ds}$ [$10^{-6}$]μs</th>
<th>Age t(days)</th>
<th>C30/37 FL 1.5/1.5</th>
<th>C30/37 FL 2.5/2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>400</td>
<td>808.0</td>
<td>805.0</td>
</tr>
<tr>
<td>FIB Model Code 2010</td>
<td>400</td>
<td>577.1</td>
<td></td>
</tr>
<tr>
<td>B3 model</td>
<td>400</td>
<td>715.1</td>
<td></td>
</tr>
<tr>
<td>B3 model improved</td>
<td>400</td>
<td>803.2</td>
<td></td>
</tr>
</tbody>
</table>

Due to the very small differences between the different types of concrete, analytical analysis of the drying shrinkage was not performed for each type.

The experimental and analytical results for the drying shrinkage up to the age of 400 days are presented in a normalized way in Figure 8.2.

![Figure 8.2: Normalized experimental and analytical results for drying shrinkage up to age of 400 days](image)

Having in mind the service life of designed structures, it is very important to be able to predict the time – dependent deformation properties for their serviceability period. Therefore, based on the results obtained for the age of up to 400 days, the analyses according to the previously mentioned models were extended to the serviceability period of the structures of 100 years. The results are presented in Figure 8.3 and in logarithmic scale in Figure 8.4. A summary of the results obtained for the age of 2, 10, 20, 50 and 100 years is given in Table 8.2.

From the figures and the table, it can be noticed that 93% of the drying shrinkage develops in the first year, 98% develops in the second year and afterwards, it reaches a final value.
Chapter 8 ANALYTICAL ANALYSIS OF THE RESULTS FROM THE EXPERIMENTAL RESEARCH

Figure 8.3: Experimental and analytical results for drying shrinkage up to age of 100 years

Figure 8.4: Experimental and analytical results for drying shrinkage up to age of 100 years in logarithmic scale

Table 8.2: Results from the analytical analysis of drying shrinkage up to age of 100 years

<table>
<thead>
<tr>
<th>Age t(years)</th>
<th>FIB MC 2010</th>
<th>B3 model</th>
<th>B3 model imp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>611</td>
<td>749.9</td>
<td>842.4</td>
</tr>
<tr>
<td>10</td>
<td>651</td>
<td>765.6</td>
<td>859.9</td>
</tr>
<tr>
<td>20</td>
<td>656</td>
<td>765.7</td>
<td>860.0</td>
</tr>
<tr>
<td>50</td>
<td>660</td>
<td>765.7</td>
<td>860.0</td>
</tr>
<tr>
<td>100</td>
<td>661</td>
<td>765.7</td>
<td>860.0</td>
</tr>
</tbody>
</table>
### 8.1.2 Analytical Analysis of Creep

Due to the differences in the creep strain between different types of concrete, an analytical analysis of the creep strain was performed for each concrete type taken separately, by the B3 model and Fib Model Code 2010.

For the concrete type C30/37, the experimental and analytical results are presented in Figure 8.5 and Table 8.3. In addition to the previously mentioned models, the improvement of the B3 model is also presented. On the basis of linear regression analysis, the following values for adjustment of the creep compliance in the B3 model were obtained: \( p_1 = 1.143 \) and \( p_2 = 1.122 \). These values are valid only for the concrete type C30/37. Taking into consideration the coefficients of variation of each model code, a good agreement was found in all cases. The Fib Model Code 2010 overestimates the experimentally obtained creep coefficient at 400 days for 13%, while the B3 model underestimates it for 9.5%.

![Figure 8.5: Experimental and analytical results for creep of C30/37 up to age of 400 days](image)

**Table 8.3: Results from the analytical analysis of instantenous and creep strain for concrete type C30/37 at the age of 400 days**

<table>
<thead>
<tr>
<th>Time-dependent properties</th>
<th>Age t(days)</th>
<th>Concrete type: C30/37</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exp.</td>
<td>FIB MC 2010</td>
</tr>
<tr>
<td>Instantaneous strain ( \varepsilon_e ) [10^{-6}]μs</td>
<td>40</td>
<td>286.3</td>
</tr>
<tr>
<td>Creep strain ( \varepsilon_{cc} ) [10^{-6}]μs</td>
<td>400</td>
<td>429.7</td>
</tr>
<tr>
<td>Inst. + creep strain ( \varepsilon = \varepsilon_e + \varepsilon_{cc} ) [10^{-6}]μs</td>
<td>400</td>
<td>716.0</td>
</tr>
<tr>
<td>Creep coefficient ( q(t,t_0) = \frac{\varepsilon_{cc}}{\varepsilon_e} )</td>
<td>400</td>
<td>1.501</td>
</tr>
</tbody>
</table>

The experimental results for both steel fibre reinforced concretes, C30/37 FL 1.5/1.5 and C30/37 FL 2.5/2.0, show a very small difference in the final strain after 400 days when the original B3 model is used. Therefore, for these types of concrete, modification of the flow compliance \( q_x \) in...
the B3 model is proposed. The modification includes addition of an amount of fibers $G_f$ to the amount of aggregate $a$, multiplied by the ratio between the moduli of elasticity of the fibers and the aggregate $E_f/E_a$.

\[ q_a = 20.3(\{(a + G_f(E_f/E_a)) / c\})^{-0.7} \] ........................................................(8.1)

With this modification, the analytically obtained final strain after 400 days was closer to the experimental one.

The results for the concrete type C30/37 FL 1.5/1.5 with 30kg/m³ steel fibers are presented in Figure 8.6 and Table 8.4, while the results for C30/37 FL 2.5/2.0 with 60 kg/m³ steel fibers are presented in Figure 8.7 and Table 8.5.

![Figure 8.6](image)

**Figure 8.6**: Experimental and analytical results for creep of C30/37 FL 1.5/1.5 up to age of 400 days

**Table 8.4**: Results from the analytical analysis of instantaneous and creep strain for concrete type C30/37 FL 1.5/1.5 at the age of 400 days

<table>
<thead>
<tr>
<th>Time-dependent properties</th>
<th>Age $t$ (days)</th>
<th>Concrete type: C30/37 FL 1.5/1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instantaneous strain $\varepsilon_e$ [$10^{-6}$]μs</td>
<td>40</td>
<td>Exp. 254.0  B3 model 280.2  B3 model mod. 280.2</td>
</tr>
<tr>
<td>Creep strain $\varepsilon_{cc}$ [$10^{-6}$]μs</td>
<td>400</td>
<td>374.7  375.8  361.6</td>
</tr>
<tr>
<td>Inst. + creep strain $\varepsilon = \varepsilon_e + \varepsilon_{cc}$ [$10^{-6}$]μs</td>
<td>400</td>
<td>628.7  656.0  641.8</td>
</tr>
<tr>
<td>Creep coefficient $\varphi(t,b) = \varepsilon_{cc} / \varepsilon_e$</td>
<td>400</td>
<td>1.475  1.341  1.291</td>
</tr>
</tbody>
</table>
Figure 8.7: Experimental and analytical results for creep of C30/37 FL 2.5/2.0 up to age of 400 days

Table 8.5: Results from the analytical analysis of instantaneous and creep strain for concrete type C30/37 FL 2.5/2.0 at the age of 400 days

<table>
<thead>
<tr>
<th>Time-dependent properties</th>
<th>Concrete type: C30/37 FL 2.5/2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Age t(days)</td>
</tr>
<tr>
<td>Instantaneous strain $\varepsilon_e [10^{-6}] \mu s$</td>
<td>40</td>
</tr>
<tr>
<td>Creep strain $\varepsilon_{cc} [10^{-6}] \mu s$</td>
<td>400</td>
</tr>
<tr>
<td>Inst. + creep strain $\varepsilon = \varepsilon_e + \varepsilon_{cc} [10^{-6}] \mu s$</td>
<td>400</td>
</tr>
<tr>
<td>Creep coefficient $\varphi(t,t_0) = \varepsilon_{cc} / \varepsilon_e$</td>
<td>400</td>
</tr>
</tbody>
</table>

The experimental and analytical results for the creep strain, up to age of 400 days, for all concrete types, are presented in a normalized way in Figure 8.8.
Figure 8.8: Normalized experimental and analytical results for creep up to age of 400 days

Based on the results obtained for an age of up to 400 days, the analyses of the creep strain according to the previously mentioned models were extended to the serviceability life of the structures of 100 years. The results for the concrete type C30/37 are presented in Figure 8.9 and, in logarithmic scale, in Figure 8.10. In Table 8.6, the results according to the Fib Model Code 2010, B3 model and B3 model improved are summarized for the age of 2, 10, 20, 50 and 100 years.

From the figures as well as from the table, it can be noticed that, unlike the drying shrinkage, creep does not reach a final value, even after 100 years. After 400 days, the creep coefficient according to the improved B3 model is 1.659. After 2 years, it is 1.867, reaching 2.810 after 100 years.

Figure 8.9: Experimental and analytical results for creep of C30/37 up to age of 100 years
Figure 8.10: Experimental and analytical results for creep of C30/37 up to age of 100 years in logarithmic scale

Table 8.6: Results from the analytical analysis of instantaneous and creep strain for concrete type C30/37 up to age of 100 years

<table>
<thead>
<tr>
<th>Time-dependent properties</th>
<th>Age t(years)</th>
<th>Concrete type: C30/37</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>FIB MC 2010</td>
</tr>
<tr>
<td>Instantaneous strain $\varepsilon_e [10^{-6}] \mu s$</td>
<td>40 days</td>
<td>278.2</td>
</tr>
<tr>
<td>Creep strain $\varepsilon_{cc} [10^{-6}] \mu s$</td>
<td>2</td>
<td>520.0</td>
</tr>
<tr>
<td>Inst. + creep strain $\varepsilon = \varepsilon_e + \varepsilon_{cc} [10^{-6}] \mu s$</td>
<td>2</td>
<td>798.2</td>
</tr>
<tr>
<td>Creep coefficient $\varphi(t,t_0) = \varepsilon_{cc} / \varepsilon_e$</td>
<td>2</td>
<td>1.869</td>
</tr>
<tr>
<td>Creep strain $\varepsilon_{cc} [10^{-6}] \mu s$</td>
<td>10</td>
<td>612.5</td>
</tr>
<tr>
<td>Inst. + creep strain $\varepsilon = \varepsilon_e + \varepsilon_{cc} [10^{-6}] \mu s$</td>
<td>10</td>
<td>890.7</td>
</tr>
<tr>
<td>Creep coefficient $\varphi(t,t_0) = \varepsilon_{cc} / \varepsilon_e$</td>
<td>10</td>
<td>2.202</td>
</tr>
<tr>
<td>Creep strain $\varepsilon_{cc} [10^{-6}] \mu s$</td>
<td>20</td>
<td>643.6</td>
</tr>
<tr>
<td>Inst. + creep strain $\varepsilon = \varepsilon_e + \varepsilon_{cc} [10^{-6}] \mu s$</td>
<td>20</td>
<td>921.8</td>
</tr>
<tr>
<td>Creep coefficient $\varphi(t,t_0) = \varepsilon_{cc} / \varepsilon_e$</td>
<td>20</td>
<td>2.313</td>
</tr>
<tr>
<td>Creep strain $\varepsilon_{cc} [10^{-6}] \mu s$</td>
<td>50</td>
<td>681.3</td>
</tr>
<tr>
<td>Inst. + creep strain $\varepsilon = \varepsilon_e + \varepsilon_{cc} [10^{-6}] \mu s$</td>
<td>50</td>
<td>959.5</td>
</tr>
<tr>
<td>Creep coefficient $\varphi(t,t_0) = \varepsilon_{cc} / \varepsilon_e$</td>
<td>50</td>
<td>2.449</td>
</tr>
<tr>
<td>Creep strain $\varepsilon_{cc} [10^{-6}] \mu s$</td>
<td>100</td>
<td>708.8</td>
</tr>
<tr>
<td>Inst. + creep strain $\varepsilon = \varepsilon_e + \varepsilon_{cc} [10^{-6}] \mu s$</td>
<td>100</td>
<td>987.0</td>
</tr>
<tr>
<td>Creep coefficient $\varphi(t,t_0) = \varepsilon_{cc} / \varepsilon_e$</td>
<td>100</td>
<td>2.548</td>
</tr>
</tbody>
</table>
The results for the concrete types C30/37 FL 1.5/1.5 and C30/37 FL 2.5/2.0 are presented in Figure 8.11 and Figure 8.13, while in logarithmic scale, they are presented in Figure 8.12 and in Figure 8.14.

Figure 8.11: Experimental and analytical results for creep of C30/37 FL 1.5/1.5 up to age of 100 years

Figure 8.12: Experimental and analytical results for creep of C30/37 FL 1.5/1.5 up to age of 100 years in logarithmic scale
Chapter 8 ANALYTICAL ANALYSIS OF THE RESULTS FROM THE EXPERIMENTAL RESEARCH

Figure 8.13: Experimental and analytical results for creep of C30/37 FL 2.5/2.0 up to age of 100 years

Figure 8.14: Experimental and analytical results for creep of C30/37 FL 2.5/2.0 up to age of 100 years in logarithmic scale

Table 8.7 provides a summary of the results obtained according to the Fib Model Code 2010, B3 model and modified B3 model for an age of 2, 10, 20, 50 and 100 years.

From the presented figures and tables, it can be noticed that the creep coefficient decreases with the increase of the residual flexural tensile strength, which is caused by the addition of fibres. After 100 years, according to the improved B3 model, the creep coefficient for the concrete without fibres, C30/37, is 2.810, while in the case of C30/37 FL 1.5/1.5 and C30/37 FL 2.5/2.0, it is 2.216 and 2.030. The decrease of the creep coefficient after 100 years is 11.1% and 17.8%, accordingly. The creep coefficient as a function of time and coefficient $\alpha_r$ that represents the ratio between the residual tensile strength and the concrete compressive strength ($\alpha_r=\sigma_r/f_{ck}$), are presented in a logarithmic scale in Figure 8.15 and in Figure 8.16 for the three concrete types.
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Table 8.7: Results from the analytical analysis of instantaneous and creep strain for concrete types C30/37 FL 1.5/1.5 and C30/37 FL 2.5/2.0 up to age of 100 years

<table>
<thead>
<tr>
<th>Time-dependent properties</th>
<th>Age t(years)</th>
<th>All concrete types</th>
<th>C30/37 FL 1.5/1.5</th>
<th>C30/37 FL 2.5/2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>FIB MC mod.</td>
<td>B3 model mod.</td>
<td>B3 model mod.</td>
</tr>
<tr>
<td>Instantaneous strain $\varepsilon_e [10^{-6}] \mu s$</td>
<td>40days</td>
<td>278.2</td>
<td>278.2</td>
<td>280.2</td>
</tr>
<tr>
<td>Creep strain $\varepsilon_{cc} [10^{-6}] \mu s$</td>
<td>2</td>
<td>520.0</td>
<td>429.6</td>
<td>409.9</td>
</tr>
<tr>
<td>Inst. + creep strain $\varepsilon = \varepsilon_e + \varepsilon_{cc} [10^{-6}] \mu s$</td>
<td>2</td>
<td>798.2</td>
<td>707.8</td>
<td>690.1</td>
</tr>
<tr>
<td>Creep coefficient $\varphi(t, t_0) = \varepsilon_{cc} / \varepsilon_e$</td>
<td>2</td>
<td>1.869</td>
<td>1.544</td>
<td>1.463</td>
</tr>
<tr>
<td>Creep strain $\varepsilon_{cc} [10^{-6}] \mu s$</td>
<td>10</td>
<td>612.5</td>
<td>531.2</td>
<td>502.2</td>
</tr>
<tr>
<td>Inst. + creep strain $\varepsilon = \varepsilon_e + \varepsilon_{cc} [10^{-6}] \mu s$</td>
<td>10</td>
<td>890.7</td>
<td>809.4</td>
<td>782.4</td>
</tr>
<tr>
<td>Creep coefficient $\varphi(t, t_0) = \varepsilon_{cc} / \varepsilon_e$</td>
<td>10</td>
<td>2.202</td>
<td>1.909</td>
<td>1.792</td>
</tr>
<tr>
<td>Creep strain $\varepsilon_{cc} [10^{-6}] \mu s$</td>
<td>20</td>
<td>643.6</td>
<td>571.2</td>
<td>538.2</td>
</tr>
<tr>
<td>Inst. + creep strain $\varepsilon = \varepsilon_e + \varepsilon_{cc} [10^{-6}] \mu s$</td>
<td>20</td>
<td>921.8</td>
<td>849.4</td>
<td>818.4</td>
</tr>
<tr>
<td>Creep coefficient $\varphi(t, t_0) = \varepsilon_{cc} / \varepsilon_e$</td>
<td>20</td>
<td>2.313</td>
<td>2.053</td>
<td>1.921</td>
</tr>
<tr>
<td>Creep strain $\varepsilon_{cc} [10^{-6}] \mu s$</td>
<td>50</td>
<td>681.3</td>
<td>623.7</td>
<td>585.4</td>
</tr>
<tr>
<td>Inst. + creep strain $\varepsilon = \varepsilon_e + \varepsilon_{cc} [10^{-6}] \mu s$</td>
<td>50</td>
<td>959.5</td>
<td>901.9</td>
<td>865.6</td>
</tr>
<tr>
<td>Creep coefficient $\varphi(t, t_0) = \varepsilon_{cc} / \varepsilon_e$</td>
<td>50</td>
<td>2.449</td>
<td>2.242</td>
<td>2.089</td>
</tr>
<tr>
<td>Creep strain $\varepsilon_{cc} [10^{-6}] \mu s$</td>
<td>100</td>
<td>708.8</td>
<td>663.3</td>
<td>621.0</td>
</tr>
<tr>
<td>Inst. + creep strain $\varepsilon = \varepsilon_e + \varepsilon_{cc} [10^{-6}] \mu s$</td>
<td>100</td>
<td>987.0</td>
<td>941.5</td>
<td>901.2</td>
</tr>
<tr>
<td>Creep coefficient $\varphi(t, t_0) = \varepsilon_{cc} / \varepsilon_e$</td>
<td>100</td>
<td>2.548</td>
<td>2.384</td>
<td>2.216</td>
</tr>
</tbody>
</table>

Figure 8.15: Creep coefficient for the three concrete types up to age of 100 years
Chapter 8 ANALYTICAL ANALYSIS OF THE RESULTS FROM THE EXPERIMENTAL RESEARCH

8.2 Analytical Analysis of Time – Dependent Deflections

There are many available analytical and numerical methods for obtaining the time – dependent response of concrete structures. However, the big number of necessary input data, complicated analysis and the big number of unknown parameters in the design phase make these methods impractical for most of the engineers.

The influence that variable load has on the time – dependent response of concrete structures is another aspect that, in certain structures, has a big effect on the total behaviour. With the experimental program of this research, it was planned to use a simple quasi – permanent load procedure to obtain a factor by which only one part of the variable load will be included in the time – dependent analysis.

8.2.1 Quasi – Permanent Load Procedure

This procedure is intended for calculation of the creep effects when serviceability limit state design should be performed using the quasi – permanent combination of actions. This combination of actions enables inclusion of the variable loads in the calculation of creep effects. One part of the variable load is added to the permanent load and is named as quasi – permanent load. The factor that defines the part of the variable load is called factor $\psi_2$ [13]. This factor depends on the category of the building and the loading history.

The loading history on different types of concrete has been the subject of research at the Faculty of Civil Engineering – Skopje for almost 12 years [21], [1]. In this research, the loading history has been chosen such that the variable load acts for 8 hours each day in the period of one year.

Four approaches to determination of the effects of variable load were considered (Fig.8.17):

1. From the point of zero value of deflection, to obtain the value of the experimentally determined deflection at the level of action of the permanent load $a_{gl(400)}$. 

Figure 8.16: Creep coefficient for the three concrete types up to age of 100 years in logarithmic scale
2. From the point of zero value of deflection, to obtain the value of the experimentally determined deflection at the level of action of the permanent and variable load $a_{g+q(400)}$.

3. From the point of the instantaneous value of deflection at permanent load level $a_{g(40)}$ (obtained at the first cycle of loading/unloading by variable load), to obtain the value of the experimentally determined deflection at action of the permanent load $a_{g(40)}$.

4. From the point of the instantaneous value of deflection at permanent load level $a_{g(40)}$, (obtained at the first cycle of loading/unloading by variable load), to obtain the value of the experimentally determined deflection at action of the permanent and variable load $a_{g+q(400)}$.

The simplest solution of the problem is taking into consideration approach 1, because the initial and time-dependent deflection can be obtained with intensity of the load as a sum of permanent and quasi-permanent load.

On the basis of the results obtained by the experimental research, an analytical solution was proposed in which the total deflection due to the permanent load “$g$” and variable load “$q$” obtained from the experiments $a_{t,exp}(g+q)$ was determined as a sum of the initial deflection $a_{i_0}(g+\psi_2q)$ and long-term deflection $a_i(g+\psi_2q)$ due to the permanent load “$g$” and variable load represented as quasi permanent load “$\psi_2q$”:

$$a_{t,exp}(g+q) = a_{i_0}(g+\psi_2q) + a_i(g+\psi_2q) \quad \text{.................................................................(8.2)}$$

This approach has one imperfection in the calculation of the instantaneous deflection, which is calculated with a lower load level.

8.2.2 Analytical Determination of Factor $\psi_2$

Factor $\psi_2$ was determined by the Age – Adjusted Effective Modulus Method (AAEMM) using the quasi – permanent load procedure and the principle of superposition. The CRACK computer program developed by Ghali A. and Elbadry M. from the University of Calgary, Canada, was used for calculation of the time – dependent deflections. This computer program was developed for...
analysis of instantaneous and time – dependent stresses and strains and the corresponding displacements in reinforced concrete members with or without prestressing. The members’ cross section must have an axis of symmetry, can be constructed and loaded in stages and can be composite or non – composite. The program accounts for the time – dependent effects of creep and shrinkage of concrete, the relaxation of prestressed steel and the effect of cracking. In the analysis, each individual section is divided into a number of concrete parts and prestressed or non – prestressed reinforcement layers, which can have different material properties. The values of the normal force and bending moment, which can vary linearly or parabolically over a specified member length, must be entered as input data.

Except the geometry, materials and bending moments, all other necessary input data needed for the AAEMM were obtained by the previously presented analysis of the time – dependent deformation properties, based on the experimentally obtained results. Those data include values which vary each day, like the values for shrinkage, creep coefficient, relaxation function and aging coefficient. For the concrete type C30/37, data obtained from the improved B3 model, while for the concrete types C30/37 FL 1.5/1.5 and C30/37 FL 2.5/2.0, data obtained from the modified B3 model, were used as input data in the AAEMM. The measurements of the time – dependent strain distribution were used for determination of the position of the neutral axis in the considered time period, taking into account the influence of the steel fibres. In absence of measurements of the efficiency factor, the overall efficiency factor of the steel fibres was taken from literature and adopted as 1/5 in the pre-cracking and 1/6 in the post-cracking stage. The application of the AAEMM in steel fibre reinforced concrete is explained in details in Chapter 6.3. The adaptation of the program for SFRC was done by inclusion of a factor by which the time – dependent moment of inertia was modified, taking into account the beneficial effect of the steel fibres.

For the concrete type C30/37, the total experimentally observed deflection was obtained with intensity of load \( g+0.37q \), which meant that factor \( \psi_2 \) had a value of 0.37. The result from the analytical analysis of the concrete type C30/37 and the comparison with the experimental one, is presented in Figure 8.18.

![Figure 8.18: Experimental and analytical time-dependent deflection for concrete type C30/37](image_url)
In the case of the steel fibre reinforced concrete types C30/37 FL 1.5/1.5 and C30/37 FL 2.5/2.0, the total experimentally observed deflection was obtained with intensity of load g+0.22q and g+0.18q, which meant that the factor $\psi_2$ had a value of 0.22 and 0.18, respectively. The results are presented in Figure 8.19 and Figure 8.20.

**Figure 8.19:** Experimental and analytical time-dependent deflection for concrete type C30/37 FL 1.5/1.5

**Figure 8.20:** Experimental and analytical time-dependent deflection for concrete type C30/37 FL 2.5/2.0

It is well known that factor $\psi_2$ depends on the duration of loading of the variable load. In the current research, the duration of the variable load was 8 hours per day and the obtained factor for the concrete type C30/37, which was without fibers, was 0.37. In the previous research projects in the same field, performed in the same laboratory with different types of concrete without fibers, the period of duration of the variable load was varied from 12h, 24h and 48h. The obtained factors $\psi_2$ in these research projects are presented in Table 8.8.
Table 8.8: Factor $\psi_2$ as a function of the duration of loading $\Delta t_{g+q}$

<table>
<thead>
<tr>
<th>$\Delta t_{g+q}$ (hours)</th>
<th>8</th>
<th>12*</th>
<th>24**</th>
<th>48**</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_2$</td>
<td>0.37</td>
<td>0.40</td>
<td>0.50</td>
<td>0.65</td>
</tr>
</tbody>
</table>

* Research of Markovski G.
**Research of Arangjelovski T.

The dependence of factor $\psi_2$ on the variable load duration can be clearly seen in the subsequent Figure 8.21.

A simple linear dependence between factor $\psi_2$ and the variable load duration can be noticed. These results are marked by the value of the ratio between the residual tensile strength and the concrete compressive strength $\alpha_f$.

For comparison, in the same figure, the values of factor $\psi_2$ for the steel fibre reinforced concretes C30/37 FL 1.5/1.5 and C30/37 FL 2.5/2.0 are also presented and marked by the ratio of $\alpha_f=0.043$ and $\alpha_f=0.063$.

For the considered variable load duration of 8h per day, the obtained factors $\psi_2$ for the three different concrete types are presented in Table 8.9. The results are presented as a function of the ratio between the residual tensile strength and the concrete compressive strength, $\alpha_f$. The dependence of factor $\psi_2$ on the ratio $\alpha_f$ is also linear and is presented in Figure 8.22.

Table 8.9: Factor $\psi_2$ as a function of the ratio between the residual tensile strength and the concrete compressive strength, $\alpha_f$

<table>
<thead>
<tr>
<th>Concrete type</th>
<th>C30/37</th>
<th>C30/37 FL 1.5/1.5</th>
<th>C30/37 FL 2.5/2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_f$</td>
<td>0</td>
<td>0.043</td>
<td>0.063</td>
</tr>
<tr>
<td>$\psi_2$</td>
<td>0.37</td>
<td>0.22</td>
<td>0.18</td>
</tr>
</tbody>
</table>
Figure 8.22: Factor $\psi_2$ as a function of the ratio between the residual tensile strength and the concrete compressive strength, $\alpha_f$
CHAPTER 9: CONCLUSIONS

From the experimental results and analytical analyses of the time-dependent behaviour of reinforced and steel fibre reinforced concrete beams subjected to permanent and repeated variable load, the following conclusions can be drawn:

1. According to the experimental results at the age of concrete of 400 days, the addition of steel fibres has almost no influence on the free drying shrinkage (2%).

2. For the considered compressive stress level, according to the experimental results at the age of concrete of 400 days, steel fibres have a bigger influence on the creep (10%). According to the B3 model, based on the experimental results, at an age of 100 years, the decrease of the creep coefficient in the case of the steel fiber reinforced concrete type C30/37 FL 1.5/1.5 is 11.1%, while in the case of C30/37 FL 2.5/2.0, it is 17.8%, when compared to ordinary reinforced concrete C30/37.

3. The analytical analysis of the drying shrinkage and creep performed by use of the B3 model and Fib Model Code 2010, demonstrated agreement with the experimental results. For concrete C30/37, an improvement of the B3 model was done on the basis of linear regression analysis. New values of coefficients $p_1$ and $p_2$ for adjustment of the creep compliance and a scaling parameter for the drying shrinkage $p_6$ were obtained. For the steel fibre reinforced concrete types, a modification of the flow compliance $q_4$ is proposed. The modification includes addition of amount of steel fibers $G_f$.

4. The repeated variable load has a significant influence on the time-dependent behaviour of the reinforced and steel fibre reinforced concrete beams.

5. Using the experimental results and analytical analysis, the following factors for quasi-permanent value of variable action were obtained: for concrete type C30/37, $\psi_2=0.37$, for concrete type C30/37 FL 1.5/1.5, $\psi_2=0.22$ and for concrete type C30/37 FL 2.5/2.0, $\psi_2=0.18$.

6. Factor $\psi_2$ depends linearly on the ratio between the residual tensile strength and the concrete compressive strength, $\alpha_f$.

7. Factor $\psi_2$ depends on the history of loading with variable load. The results obtained through this research as well as through similar previous research projects performed at the Faculty of Civil Engineering in Skopje, demonstrated a linear dependence between factor $\psi_2$ and variable load duration.

8. For further research, different mechanical reinforcement ratios and stress levels as well as “random” variable load duration should be used in order to obtain more generalized dependencies.
REFERENCES

[31] Test and Design Methods for Steel Fibre Reinforced Concrete - Background and Experiences- Proceedings of the RILEM TC 162 - TDF Workshop - Edited by B.Schnütgen, L.Vandewalle.